Lesson 7: Watch Your Step!

In previous lessons, we have looked at techniques for solving equations, a common theme throughout algebra. In this lesson, we examine some potential dangers where our intuition about algebra may need to be examined.

Opening Exercise

1. Describe the property used to convert the equation from one line to the next:

$$x(1-x)+2x-4=8x-24-x^2$$

$$x-x^2+2x-4=8x-24-x^2 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

$$x+2x-4=8x-24 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

$$3x-4=8x-24 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

$$3x+20=8x \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

$$20=5x \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

In each of the steps above, we applied a property of real numbers and/or equations to create a new equation.

2. Why are we sure that the initial equation $x(1-x)+2x-4=8x-24-x^2$ and the final equation $20=5x$ have the same solution set?

3. What is the common solution set to all these equations?

4. Solve the equation for x. For each step, describe the operation used to solve the equation.

$$3x - [8 - 3(x - 1)] = x + 19$$

Work	Reasoning
$3x - [8 - 3(x - 1)] = x + 19$	Given

5. Consider the equation $\frac{1}{x} = \frac{3}{x-2}$.

$3(x) = 1(x-2)$
$3x = x - 2$
$-x \quad -x$
$2x = -2$ $\boxed{x = -1}$

A. What values of *x* would lead to division by 0? Why is division by 0 a problem?

$\frac{1}{-1} = \frac{3}{-1-2}$

$-1 = \frac{3}{-3}$ $\quad -1 = -1$

B. Thinking back on the work you've done in this module. What could you do to isolate the variable?

C. Avery rewrote this equation as:

$$(x-2)(x)\left(\frac{1}{x}\right)=(x-2)(x)\left(\frac{3}{x-2}\right)$$

$$(x-2)(1)=(x)(3)$$

Explain what Avery did in his first two steps and then finish finding the solution.

6. Consider the equation $\frac{3}{x-2}=\frac{5}{x-2}$.

A. What values of x would lead to division by 0?

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B. Use Avery's method to find the value of x.

$x=2$

C. How could you tell by looking at the original equation that the solution was going to be different?

$$\frac{3}{x-2}=\frac{5}{x-2}$$

$5(x-2)=3(x-2)$

$5x-10=3x-6$

$-3x \quad -3x$

$2x-10=-6$

$+10 \quad +10$

$2x=4$

$x=2$

When solving equations that have a variable in the denominator it is critical that you ***exclude*** values that would make the denominator equal to zero.

Determine the excluding the value(s) of x that lead to a denominator of zero for each equation; then, solve the equation for x.

7. $\frac{5}{x} = 1$

$x \neq 0$

$\frac{5}{x} = \frac{1}{1}$

$\cancel{x} \cdot \frac{5}{\cancel{x}} = 1 \cdot x$

$5 = x$

$\boxed{x = 5}$

8. $\frac{1}{x-5} = 3$

$x \neq 5$

$\frac{1}{x-5} = \frac{3}{1}$

$3(x-5) = 1$

$3x - 15 = 1$

$+15 \quad +15$

$3x = 16$

$\boxed{x = \frac{16}{3}}$

9. $\frac{x+1}{x+1} = 4$

$x \neq -1$

$\frac{x+1}{x+1} = \frac{4}{1}$

$4(x+1) = 1(x+1)$

$4x + 4 = x + 1$

$-x \quad -x$

$3x + 4 = 1$

$-4 \quad -4$

$3x = -3$

$x = -1$

No Solution

10. $\frac{2}{x} = \frac{3}{x-4}$

$x \neq 4 \quad x \neq 0$

$2(x-4) = 3x$

$2x - 8 = 3x$

$-2x \quad -2x$

$\boxed{-8 = x}$

11. $\frac{3}{x+6} = \frac{-6}{x+6}$

$x \neq -6$

$3(x+6) = -6(x+6)$

$3x + 18 = -6x - 36$

$+6x \quad +6x$

$9x + 18 = -36$

$-18 \quad -18$

$9x = -54 \quad x = -6$

No Solution

12. $\frac{x-3}{x+2} = 0$

$x \neq -2$

$\frac{x-3}{x+2} = \frac{0}{1}$

$1(x-3) = 0(x+2)$

$x - 3 = 0$

$\boxed{x = 3}$

Lesson Summary

Applying the distributive, commutative, and associative properties and the properties of equality to equations will not change the solution set.

When solving equations be careful to exclude any solutions that would make the denominator equal to 0.

The equation $\frac{2x}{x-2} = 4$ has an excluded value of 2.

13. Determine the solution to the Lesson Summary.

$\frac{2x}{x-2} = \frac{4}{1}$

$1(2x) = 4(x-2)$

$2x = 4x - 8$

$-4x \quad -4x$

$-2x = -8$

$\boxed{x = 4}$

Homework Problem Set

1. Solve for x and fill in the reasons for each step.

$\frac{1}{5}[10-5(x-2)] = \frac{1}{10}(x+1)$	Original statement
$2[10-5(x-2)] = (x+1)$	Multiply both sides by ___________.
$2[10-5x+10] = (x+1)$	
$2[20-5x] = (x+1)$	
$40-10x = x+1$	
$40 = 11x+1$	
$39 = 11x$	
$\frac{39}{11} = x$	

Solve each equation for x. Be sure to show each step, but you do not need to give a reason for each one.

2. $x + 6 - x = 2x + 10$	3. $15 = \frac{3}{5}x$	4. $5(x+5) = 10$
5. $x + 11 + x = -7$	6. $2x + 7 = 4x - 9$	7. $5x + 4 = 4x + 4$
8. $9(x+4) = 9x + 4$	9. $3x - 7 + 5 = 2(x-2)$	10. $2x + 9 + x = 3(x-2) + 15$

11. Consider the equation $\frac{10(x^2-49)}{3x(x^2-4)(x+1)} = 0$. Is $x = 7$ permissible? Which values of x are excluded? (You do not need to solve this equation.)

Determine the excluding the value(s) of x that lead to a denominator of zero for each equation; then, solve the equation for x.

12. $\frac{2}{x} = \frac{5}{x+1}$	13. $\frac{1}{5x} = 10$

14. $\frac{x+3}{x+3} = 5$	15. $\frac{x+3}{x+3} = 1$

Solve each equation for x.

16. $7x - [4x - 3(x - 1)] = x + 12$

17. $2[2(3 - 5x) + 4] = 5[2(3 - 3x) + 2]$

18. $\frac{1}{2}(18 - 5x) = \frac{1}{3}(6 - 4x)$

19. $18 = \frac{2}{3}x$

20. **CHALLENGE** Write an equation with the restrictions $x \neq 14$, $x \neq 2$, and $x \neq 0$.

21. **CHALLENGE** Write an equation that has no solution.

22. **CHALLENGE** Use any of the digits 1 – 9 to create an equation with the smallest solution possible.

$$\frac{\square}{\square}x - \square = \square$$

CHALLENGE Determine the excluded value for each equation. You do **NOT** need to solve the equation.

23. $\dfrac{3}{x-7}=5$

24. $-4=\dfrac{3}{x+4}$

25. $\dfrac{(x-2)(x+1)}{(x-1)(x+1)}=7$

26. $\dfrac{(x-3)}{(x-3)(x+4)}=\dfrac{(x+4)}{(x+4)}$

27. $10=\dfrac{(x+3)(x+5)}{(x+5)(x+6)}$

28. $-2=\dfrac{4-x}{6}$