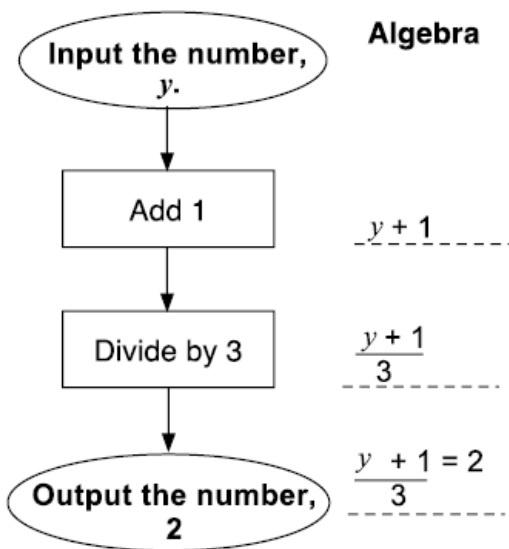# Lesson 4: Building and Solving Equations

Opening Exploratory – Adapted from MARS: Building and Solving Linear Equations

1. Here is an algebra machine. The Algebra column shows what happens to the unknown.
    A. Solve the equation. Be sure to show and explain all your steps.

Algebra

Input the number, y.

Add 1 $y + 1$

Divide by 3 $\dfrac{y + 1}{3}$

Output the number, 2 $\dfrac{y + 1}{3} = 2$

Use this space to solve the equation $\dfrac{y+1}{3} = 2$

$$3 \cdot \frac{y+1}{3} = 2 \cdot 3$$

$$y+1 = 6$$

$$\frac{-1 \quad -1}{y = 5}$$

B. How can you check your solution? Is it correct?

Plug it in $\dfrac{5+1}{3} = 2$ $\dfrac{6}{3} = 2$

C. How can "undoing" help you solve this problem?

2. Here is another algebra machine.

A. Complete the Algebra column. Solve your equation. Show and explain all your steps.

Use this space to solve the equation.

$$5 \cdot \frac{3(x+7)-2}{5} = 11 \cdot 5$$

$$3(x+7)-2 = 55$$
$$\qquad +2 \qquad +2$$
$$\frac{3(x+7)}{3} = \frac{57}{3}$$

$$x+7 = 19$$
$$-7 \quad -7$$
$$x = 12$$

Input the number, x.

Add 7 $x+7$

Multiply by 3 $3(x+7)$

Subtract 2 $3(x+7)-2$

Divide by 5 $\dfrac{3(x+7)-2}{5}$

Output the number, 11 $\dfrac{3(x+7)-2}{5} = 11$

B. When the 3rd step says to multiply by 3, it means multiply the whole expression by 3. How did you write this?
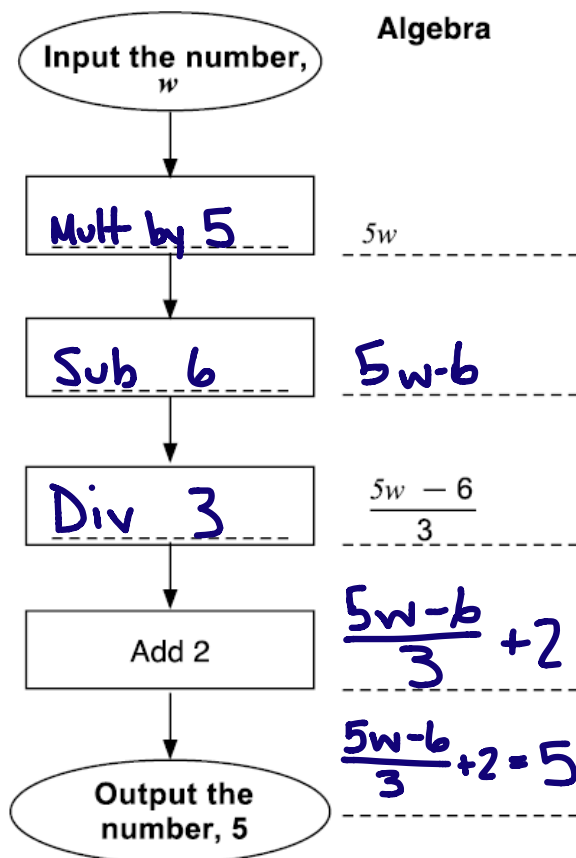
We used parentheses

C. In the 5th step, you are to divide by 5. What exactly is divided by 5?

The whole expression

D. Since the output is 11, we know the whole expression up to this point is equal to 11. Why is this necessary to find the value of x?

3. Here is a third algebra machine.

    A. Complete the machine instructions and the Algebra column. Solve your equation. Show and explain all your steps.

Algebra

Input the number, w

Mult by 5 $5w$

Sub 6 $5w-6$

Div 3 $\dfrac{5w-6}{3}$

Add 2 $\dfrac{5w-6}{3}+2$

Output the number, 5 $\dfrac{5w-6}{3}+2=5$

Use this space to solve the equation.

    B. This algebra machine was the most difficult because the machine instructions were missing in parts. How did you determine the 3rd step where both the machine instructions and the Algebra column were blank?

4. Amy has correctly built an equation and has not made any mistakes, but she is missing the explanation of each step she took to create her equation. Write in Amy's steps.

$$x = 6$$

$$x + 2 = 8$$

$$\frac{x + 2}{2} = 4$$

$$\frac{x + 2}{2} - 1 = 3$$

$$3\left(\frac{x + 2}{2} - 1\right) = 9$$

5. Laura has used arrows to connect the steps she has taken and she has shown each step. However, she made a mistake when building her equation. Where is Laura's mistake?

$$x = 6$$

$$-2 \, \curvearrowright \quad x - 2 = 4$$

$$\div 4 \, \curvearrowright \quad \frac{x-2}{4} = 1$$

$$+3 \, \curvearrowright \quad \frac{x+1}{4} = 4$$

$$\times 5 \, \curvearrowright \quad \frac{5(x+1)}{4} = 20$$

6. Below is Mason's final equation. Show how Mason built this equation from $x = 6$.

$$\frac{2x + 2}{5} = 2 \cdot 8$$

7. Liam had the final equation $\frac{3(2a - 1)}{7} = 6$, but he forgot to tell what value of a he started with. Use the ideas from this lesson to determine the value of a.

Clearing Fractions

In each problem in this lesson we had a ***rational expression*** (fraction) on one side of the equal sign. We were able to "undo" the fraction by multiplying by the denominator on both sides of the equation. Let's look at some equations that have more than one rational expression.

8. For the equation below, what is the least common denominator (LCD)?

> What number is a multiple of both 6 and 8?

A. Multiply this number on both sides of the equation and solve for *x*.

$$\frac{x}{6} = \frac{x+1}{8}$$

> You will have to distribute on the right side of the equation.

$$24\left(\frac{x}{6}\right) = 24\left(\frac{x+1}{8}\right)$$

$$\frac{24}{6} \cdot x = \frac{24}{8}(x+1)$$

$$4x = 3(x+1)$$

B. Check your solution.

9. Solve $\frac{x+4}{2} = \frac{-2x+7}{6}$. Does your answer make sense?

$$6\left(\frac{x+4}{2}\right) = 6\left(\frac{-2x+7}{6}\right)$$

$$3(x+4) = -2x+7$$

$$3x+12 = -2x+7$$

$$3x+12 = -2x+7$$
$$+2x \qquad +2x$$
$$\overline{5x+12 = 7}$$
$$-12 \quad -12$$
$$\overline{\frac{5x}{5} = \frac{-5}{5}}$$
$$\boxed{x = -1}$$

10. You'll do the same procedure with this equation: $\dfrac{x}{5}+\dfrac{x}{10}=\dfrac{x+2}{5}$. Check that your solution is correct.

$$\frac{x}{5}+\frac{x}{10}=\frac{x+2}{5}$$

Lesson Summary

To solve equations:

Use the **commutative, associative, distributive properties**
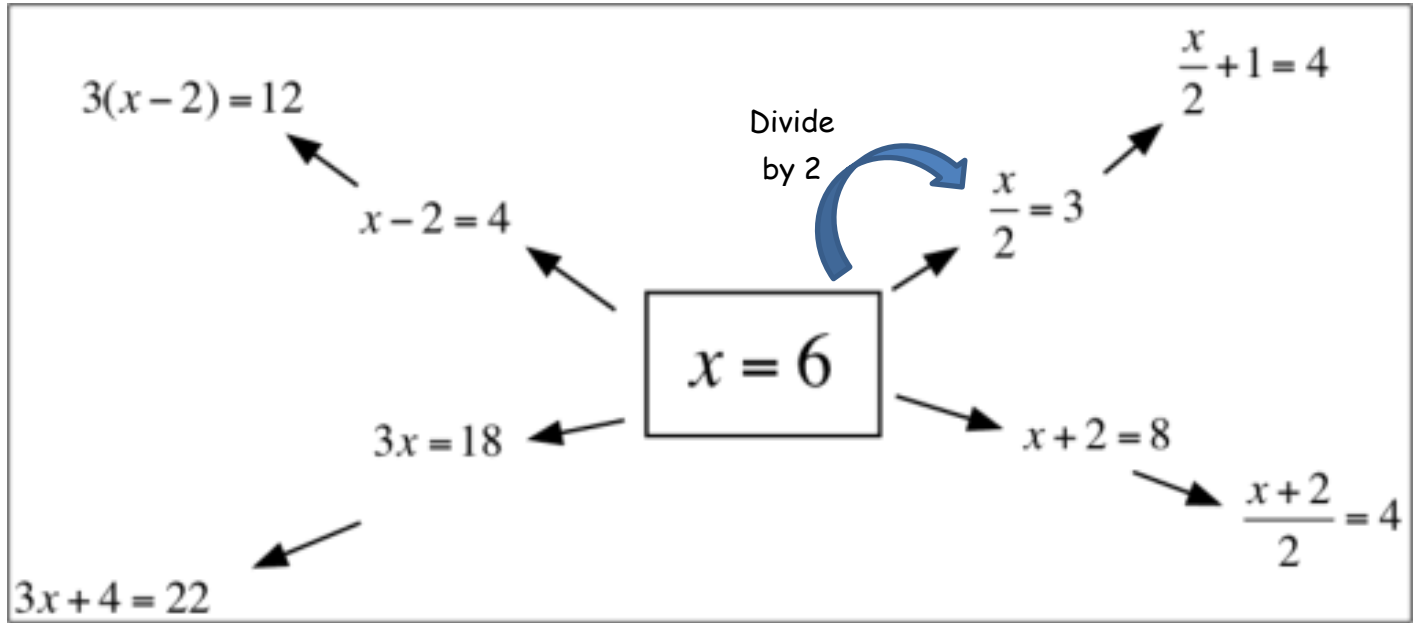
AND

Use the **properties of equality**

(adding, subtracting, multiplying by non-zeros, dividing by non-zeros)

When faced with rational expressions in your equation, think about **clearing the fractions** by multiplying both sides by the Least Common Denominator.

Homework Problem Set

1. For each branch below, write in the steps that were used to get to the final equation. One step was done for you.

The diagram shows $x = 6$ in the center box with branches:

- $3(x-2) = 12$
- $x - 2 = 4$
- $3x = 18$
- $3x + 4 = 22$
- $\dfrac{x}{2} = 3$ (labeled "Divide by 2")
- $\dfrac{x}{2} + 1 = 4$
- $x + 2 = 8$
- $\dfrac{x+2}{2} = 4$

2. A. Determine the correct order of the equation that is being built from $a = 4$.   __C__ ____ ____ ____ __F__

A.	$\dfrac{3(2a-1)}{4} = 5.25$	B.	$2a = 8$
C.	$a = 4$	D.	$3(2a -1) = 21$
E.	$2a -1 = 7$	F.	$\dfrac{3(2a-1)}{4}+1 = 6.25$

B. Determine the correct order of the steps.   __H__ ____ ____ ____ ____

G.	Subtract 1 from each side.	H.	Original statement.
I.	Add 1 to both sides.	J.	Divide each side by 4.
K.	Multiply each side by 3.	L.	Multiply both sides by 2.

Determine the value of the variable in each equation. Think about how the equation was built and how you could "undo" it. Be sure to check that your solution makes the equation true.

3. $x + 5 = 11$

4. $2x = 16$

5. $2x + 5 = 25$

6. $3x - 13 = 5$

7. $-4x = 24$

8. $-x + 1 = 12$

9. $-x + 3 = -4$

10. $5x - 2 = 48$

11. $6x - 12 = 0$

12. $\dfrac{1}{2}x = 7$

13. $\dfrac{1}{4}x - 1 = 4$

14. $x + \dfrac{1}{3} = \dfrac{2}{3}$

15. $\dfrac{x}{3} = 8$

16. $\dfrac{x}{5} = -2$

17. $x - \dfrac{1}{4} = \dfrac{3}{4}$