Lesson 3: Solving Equations – A Balancing Act

Opening Exercise

Let's look back at the puzzle in Lesson 1 with the *t*-shape and the 100-chart. Jennie came up with a sum of 380 and through the lesson we found that the expression to represent the sum of the five numbers was $5x + 50$ where x is the smallest of the five numbers. This gives us the equation $5x + 50 = 380$. By the end of this lesson, we'll be able to solve Jennie's equation.

We'll begin by thinking of solving equations as a balance scale.

1. What equation could the following picture represent?

[Image Source: http://www.eduplace.com]

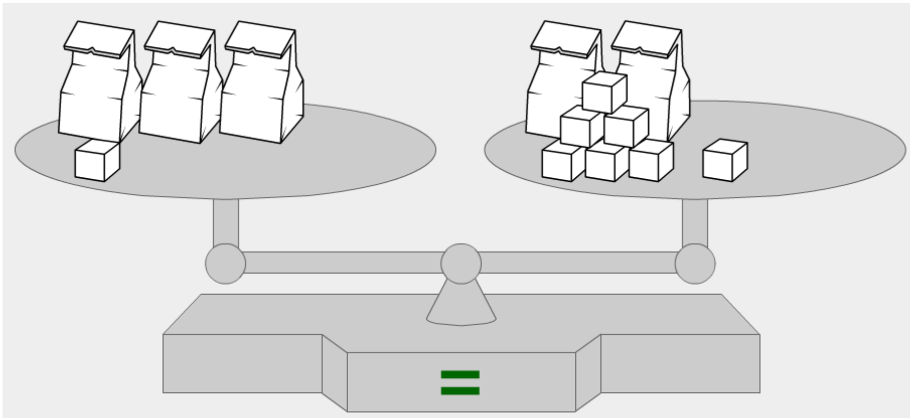
2. If the scale must be kept balanced, whatever is removed from one side of the scale must be removed from the other side of the scale.

   A. How can you get the X alone?

   B. What is the value of X?

   C. How can you represent the original problem and its solution using an equation?

3. In the following picture, we are trying to determine how many blocks are in one bag.



A. How can you get one bag only on the scale?

Take away 2 bags on each side.

B. How many blocks are in one bag?

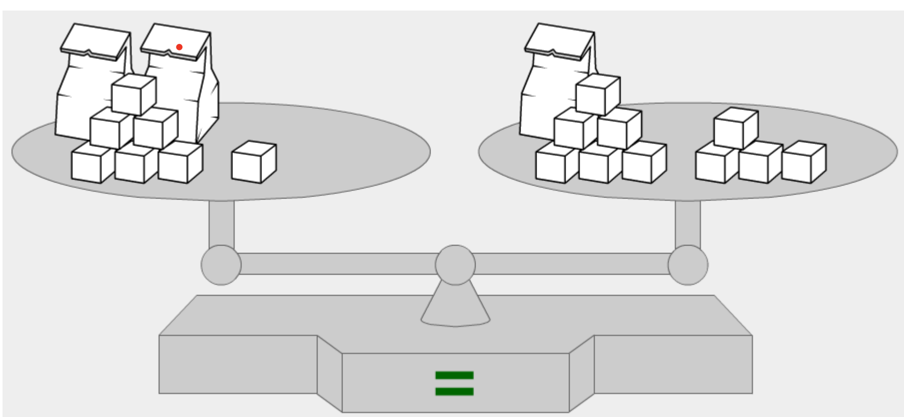
6 blocks

C. How can you represent the original problem and its solution using an equation?

$3x+1=2x+7$
$-2x \quad -2x$

$x+1=7$
$x=6$

4. Write an equation and solve it to find the number of blocks in one bag for the problem illustrated below.

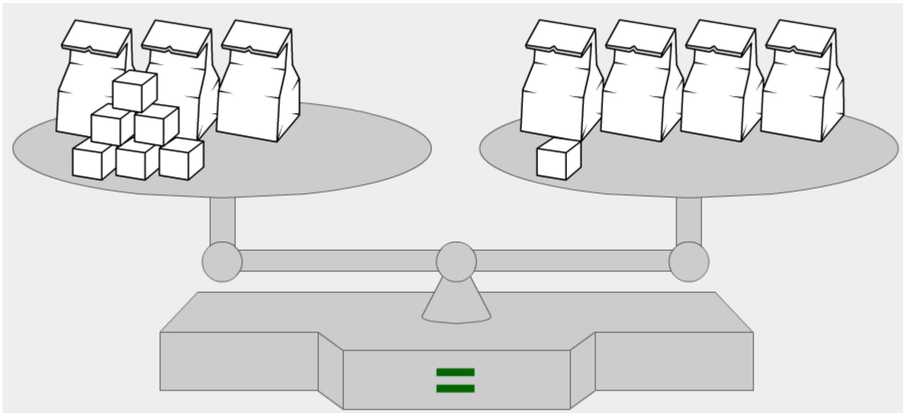


$2x+7=x+10$
$-x \quad -x$

$x+7=10$

$\boxed{x=3}$

5. Write an equation and solve it to find the number of blocks in one bag for the problem illustrated below.



$3x+6=4x+1$

$-3x \quad -3x$

$6=x+1$

$\boxed{5=x \quad x=5}$

6. A. Write the following problem as a balance problem, using bags and blocks.

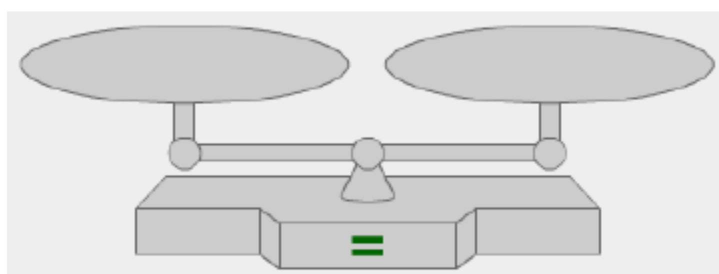
$$x+8=3x+4$$

$-x \quad -x$

$8=2x+4$

$4=2x$

$\boxed{x=2}$



B. How many blocks are in one bag?

2 blocks

[source: https://ca.pbslearningmedia.org/resource/mgbh.math.ee.balance/balancing-scales-to-solve-equations/#.WZtx2mendGY]

Exploratory Activity

In elementary school you learned that 3 + 4 gives the same answer as 4 + 3 and that 3 • 4 gives the same answer as 4 • 3. These are both examples of the ***commutative property*** – one for addition and one for multiplication.

7. In the chart below, circle all the statements that are showing commutative property for multiplication and underline all the statements showing the commutative property for addition.

$x \bullet 1 = x$	$(a+y)(b+x) = (b+x)(a+y)$	$(xy)z = x(yz)$	$(ab)+c = c+(ab)$
$(a+b)+c = a+(b+c)$	7 • 8 = 8 • 7	$(7 \bullet 8) \bullet 9 = 7 \bullet (8 \bullet 9)$	$x \bullet y = y \bullet x$
$a(b+c) = ab+ac$	$(7+8)+9 = 7+(8+9)$	$x+0 = x$	$x+y = y+x$
$(ab) \bullet c = c \bullet (ab)$	$x \bullet \frac{1}{x} = 1,\ x \neq 0$	$a(b-c) = ab-ac$	8 + 7 = 7 + 8

8. In your own words, explain what the commutative property does.

Changes the order.

Another important rule from elementary school was the associative property. This one deals with how numbers or variables are grouped. You learned that (3 + 4) + 5 will give the same answer as 3 + (4 + 5). This is the associative property with respect to addition. The associate property with respect to multiplication is similar (3 • 4) • 5 = 3 • (4 • 5).

9. Are all the rest of the statements, using the associative property? Put a star (*) next to any that are not showing either the associative or the commutative property.

10. The statements $a(b+c)=ab+ac$ and $a(b-c)=ab-ac$ are examples of the distributive property. Choose any three numbers for a, b and c to see that both statements are true.

a = 5, *b* = 2, *c* = 24

$a(b+c)=ab+ac \rightarrow$ 5(2+24) = 5(2)+5(24) and
130 10+120=130

$a(b-c)=ab-ac \rightarrow$ 5(2-24) = 5(2)-5(24)
-110 10-120=-110

We saw in Exercise 10, that no matter what three numbers we choose, the two sides of each statement would have to be equal. We'll use the distributive property in greater depth in Lesson 5.

Here's the strategy:

If we are faced with the task of solving an equation, that is, finding the solution set of the equation:

Use the commutative, associative, distributive properties

AND

Use the properties of equality

(adding, subtracting, multiplying by non-zeros, dividing by non-zeros)

to keep rewriting the equation into one whose solution set you easily recognize. (We observed that the solution set <u>will not change</u> under <u>these</u> operations.) This usually means rewriting the equation so that all the terms with the variable appear on one side of the equation.

11. To illustrate this idea, in this exercise you and your team will solve the equation $3x + 4 = 8x - 16$ starting in four different ways. Determine who will be Student 1, 2, 3 and 4. Then solve for x using the given starting point.

$$3x + 4 = 8x - 16$$

Student 1	**Student 2**	**Student 3**	**Student 4**
Subtract $3x$ from both sides	*Subtract 4 from both sides*	*Subtract $8x$ from both sides*	*Add 16 to both sides*

12. Did everyone get the same value for *x*? If not, check to see where a mistake was made. Remember you must do the same thing to both sides of the equation to keep it balanced.

13. We are ready to solve Jennie's equation using these ideas. Write each step as you solve Jennie's problem. Was Jonathan correct when he said her smallest number was 66?

$$\mathbf{5x + 50 = 380}$$

Lesson Summary

If x is a solution to an equation, it will also be a solution to the new equation formed when the same number is added to (or subtracted from) each side of the original equation or when the two sides of the original equation are multiplied by (or divided by) the same non-zero number. These are referred to as the ***properties of equality***.

If you are faced with the task of solving an equation, that is, finding the solution set of the equation:

- Use the ***commutative***, ***associative***, and ***distributive properties***, AND use the ***properties of equality*** (adding, subtracting, multiplying by non-zeros, dividing by non-zeros) to keep rewriting the equation into one whose solution set you easily recognize.

Homework Problem Set

1. Alonzo was correct when he said the following equations had the same solution. Why was Alonzo correct?

$$(x-1)(x+3) = 17 + x \qquad\qquad (x-1)(x+3) = x + 17$$

2. He then said that $(x-1)(x+3) = 17 + x$ and $(x-1)(x+3) + 500 = 517 + x$ should have the same solution set. Is he correct? Explain your reasoning.

3. Finally, Alonzo said the equations $(x-1)(x+3) = 17 + x$ and $3(x-1)(x+3) = 51 + 3x$ should have the same solution set. What do you think? Why?

4. Consider the equation $x^2 + 1 = 7 - x$.
   a. Verify that this has the solution set $\{-3, 2\}$.

   b. Let's add 4 to both sides of the equation and consider the new equation $x^2 + 5 = 11 - x$. Verify 2 and -3 are still solutions.

   c. Let's now add x to both sides of the equation and consider the new equation $x^2 + 5 + x = 11$. Are 2 and -3 still solutions?

   d. Let's add -5 to both sides of the equation and consider the new equation $x^2 + x = 6$. Are 2 and -3 still solutions?

   e. Let's go back to part (d) and add $3x^3$ to both sides of the equation and consider the new equation $x^2 + x + 3x^3 = 6 + 3x^3$. Are 2 and -3 still solutions?

5. Let's go back to Exercise 4 and this time multiply both sides by $\frac{1}{6}$ to get $\frac{x^2+x}{6} = 1$. Are 2 and -3 still solutions?

Solve each equation and check your solution.

[source: Kuta Software]

6. $-20 = -4x - 6x$

7. $6 = 1 - 2n + 5$

8. $8x - 2 = -9 + 7x$

9. $a + 5 = -5a + 5$

10. $4m - 4 = 4m$

11. $p - 1 = 5p + 3p - 8$

12. $5p - 14 = 8p + 4$

13. $b - 4 = -9 + b$

14. $12 = 24x + 12$

15. $-18 - 6k = 6 + 18k$

16. $-7+4x=9$

17. $5n+34=-2+14n$

18. $8x-6-8=4+2x$

19. $3n-5=-48-40n$

20. $24a-22=-4+24a$

21. $-12x-9+24x+4=43$

22. $-1-7x+42+6x=36$

23. $-5+25x-40x-10=-4x-8x$

24. $-7+4x=9$

25. $-8=-x-4$