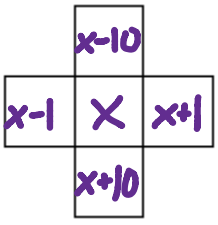
Lesson 1: The Science of Patterns

Exploratory Exercise - [adapted from NCTM Illuminations]

You will need: a *t*-shape handout

1. Jennie used a *t*-shape (shown below) to cover five numbers. The sum of her numbers was 380. Jonathan said, "I know your smallest number is 66." How did Jonathan figure out Jennie's smallest number?



$x-10+x-1+x+x+1+x+10$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2. We can determine Jonathan's trick by looking for patterns in the numbers the *t*-shape covers. Use the 100-chart below and your *t*-shape to cover five of the numbers. Write down your five numbers and then note at least 4 observations about these numbers.

My five numbers:

66 75 76 77 86

My observations:

1.

2.

3.

4.

3. Gather data from at least three other students about their five numbers. Do your observations hold for each of their number choices?

Five Numbers	Sum of Five Numbers	Observations that are still true

4. Let n be the **least** number of your group of five numbers. Write an expression for each of your five numbers based on n.

5. A. What is the sum of your five numbers?

B. How can your expressions from Exercise 3 be used to verify that sum?

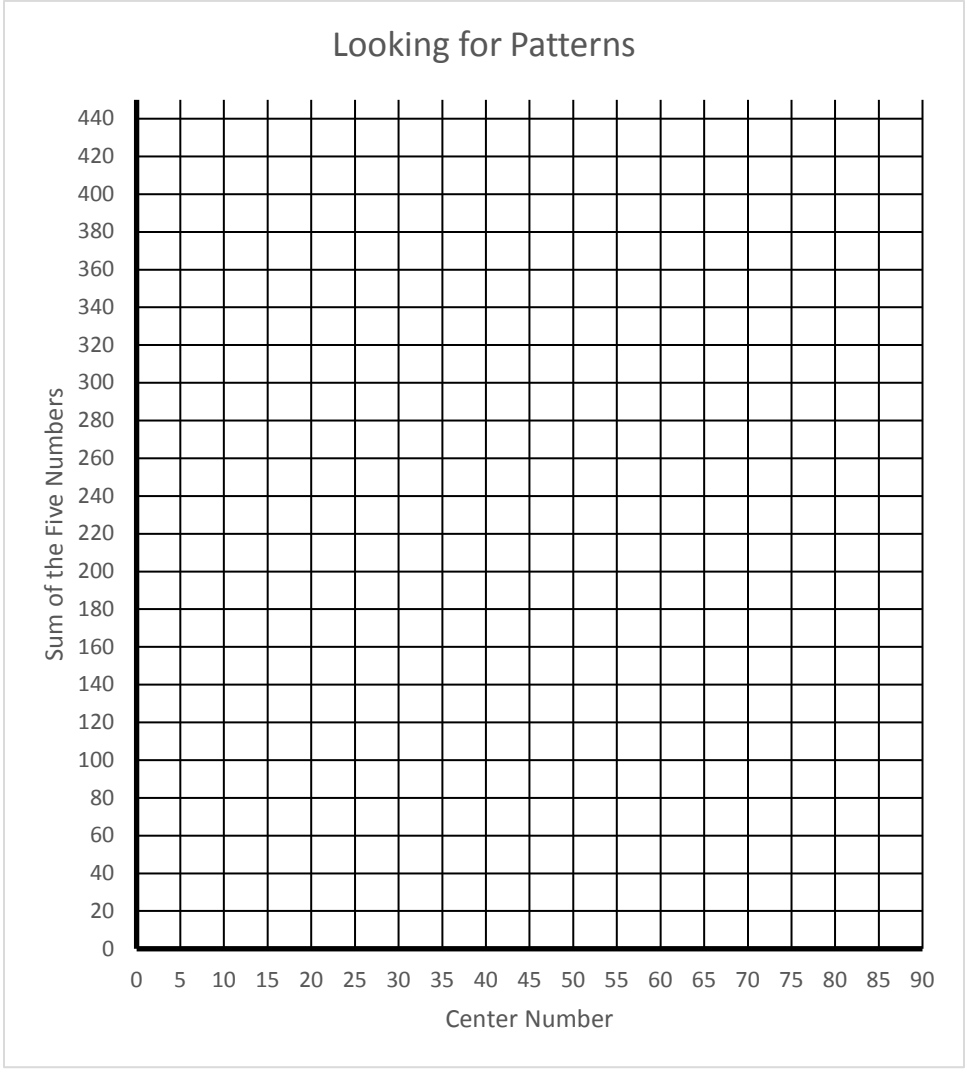
6. Melissa says that the sum of her five numbers is 65. Write an equation that could be used to find the smallest of Melissa's five numbers. What is the smallest of Melissa's five numbers?

7. Troy's sum is 14. Explain why this is impossible if the *t*-shape is placed properly.

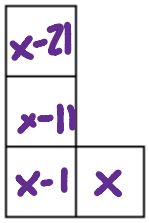
Looking at Patterns with Graphs

8. A. On the graph at the right, plot the **center** number on the horizontal axis and the **sum** of the five numbers on the vertical axis. Gather more data from other students or find your own sums.

   B. What patterns do you notice?



9. Sunny states that you can find a different pattern if you use an "L" shape, as shown on the right. Use Sunny's L-shape to write a new expression for the sum of the four numbers, with *n* representing the smaller number of the four.



$x + x-1 + x-11 + x-21$

$4x - 33 = 71$

$+33 \quad +33$

$4x = 104$

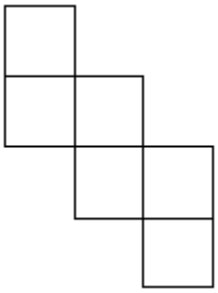
$x = 26$

25

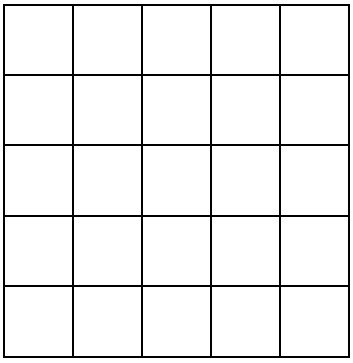
15

5

10. Angel used a zig-zag pattern to write his expression. If *n* is the largest number in Angel's group of six numbers, what is the expression for the sum of the six zig-zag numbers?



11. Use the grid at the right to create your own shape pattern. Then write an expression for the sum of the numbers in your pattern. Be sure to state which number your variable represents (smallest, largest, middle).



Homework Problem Set

1. There are 29 students in Miss Spelling's class. As a special holiday gift, she bought each of them chocolate letters with which they can spell their names. Unfortunately, some letters cost more than others – for instance, the letter A, which is in high demand, is rather pricey; whereas the letter Q, which almost no one needs, is relatively inexpensive.

   The price of the chocolate letters for each student in her class is shown in the table below.

AIDEN = 386	ARI = 209	ARIEL = 376	BLAIRE = 390	CHARLES = 457
CLARE = 334	DEAN = 317	EARL = 307	FRIDA = 273	GABRIEL = 410
IVY = 97	KOLE = 249	LEIA = 317	LEO = 242	MAVIS = 246
NADINE = 453	NED = 236	PAUL = 167	QASIM = 238	RACHEL = 394
RAFI = 231	SAM = 168	TIRA = 299	ULA = 148	VERA = 276
VIJAY = 179	WOLKE = 272	XAVIER = 346	ZERACH = 355	

   A. You can use groups of letters to determine the value of each letter. For example, ARI = 209, and ARIEL = ARI + EL. We can then write an equation combining these two statements.

   $$\text{ARIEL} = \text{ARI} + \text{EL}$$

   $$376 = 209 + \text{EL}$$

   $$\text{EL} = 167$$

   Write at least three different equations of letter combinations.

   B. Were there any single letter values you found? What were they?

[source: NCTM Illuminations]

Write each as an algebraic expression. [source: Kuta software]

2. the difference of 10 and 5

3. the quotient of 14 and 7

4. u decreased by 17

5. half of 14

6. x increased by 6

7. The product of x and 7

8. the sum of q and 8

9. 6 squared

Write each as a verbal expression. [source: Kuta software]

10. $a + 9$

11. $19 - 3$

12. $5n$

13. q^2

14. $\frac{a}{8}$

15. $x + 8$

16. $n \bullet 6$

17. 2^2

Evaluate each expression. [source: Kuta software]

18. 5 squared

19. The product of 8 and 10

20. 20 decreased by 17

21. The quotient of 96 and 8