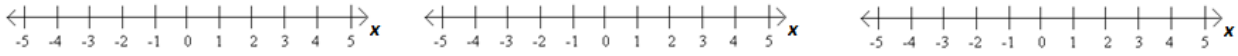
Lesson 12: Absolute Value Inequalities

Warm-Up Exercise

1. Use the number lines below to graph each inequality.

A. $x < 4$ B. $x \geq 1$ C. $2 + x \leq 5$



Exploratory Exercise

Next, we'll combine two ideas – inequalities and absolute value.

2. For each inequality below, think about all the points that would make the inequality true and plot those points. What type of endpoint should each inequality have? Are there other points that would also make the inequality true?

A. $|x| < 4$ B. $|x| \geq 1$ C. $|2 + x| \leq 5$

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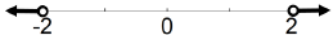
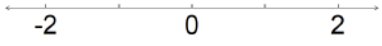
3. If we think about the meaning of absolute value as a distance from zero it leads us to a strategy that doesn't involve guessing. Consider these examples:

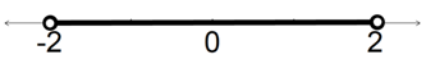
	$\lvert x\rvert > 2$	$\lvert x\rvert < 2$	$\lvert x\rvert > -2$
Describe the meaning	All the numbers which are greater than 2 units away from zero.	All the numbers which are _____ than ___ units away from zero.	All the numbers which are _____ than ___ units away from zero.
Represent the solution set on a number line			
Write inequalities to describe the solution set	$x >$ ___ or $x <$ ___		

4. Work with your partner to decide where the following examples fit into Tanya's flow chart on the next page.

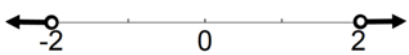
No solution.

$-2 < x < 2$

$x > 2$ or $x < -2$



All real numbers.



$\lvert x\rvert > 2$

$\lvert x\rvert > 2$

$\lvert x\rvert > -2$

$\lvert x\rvert > -2$

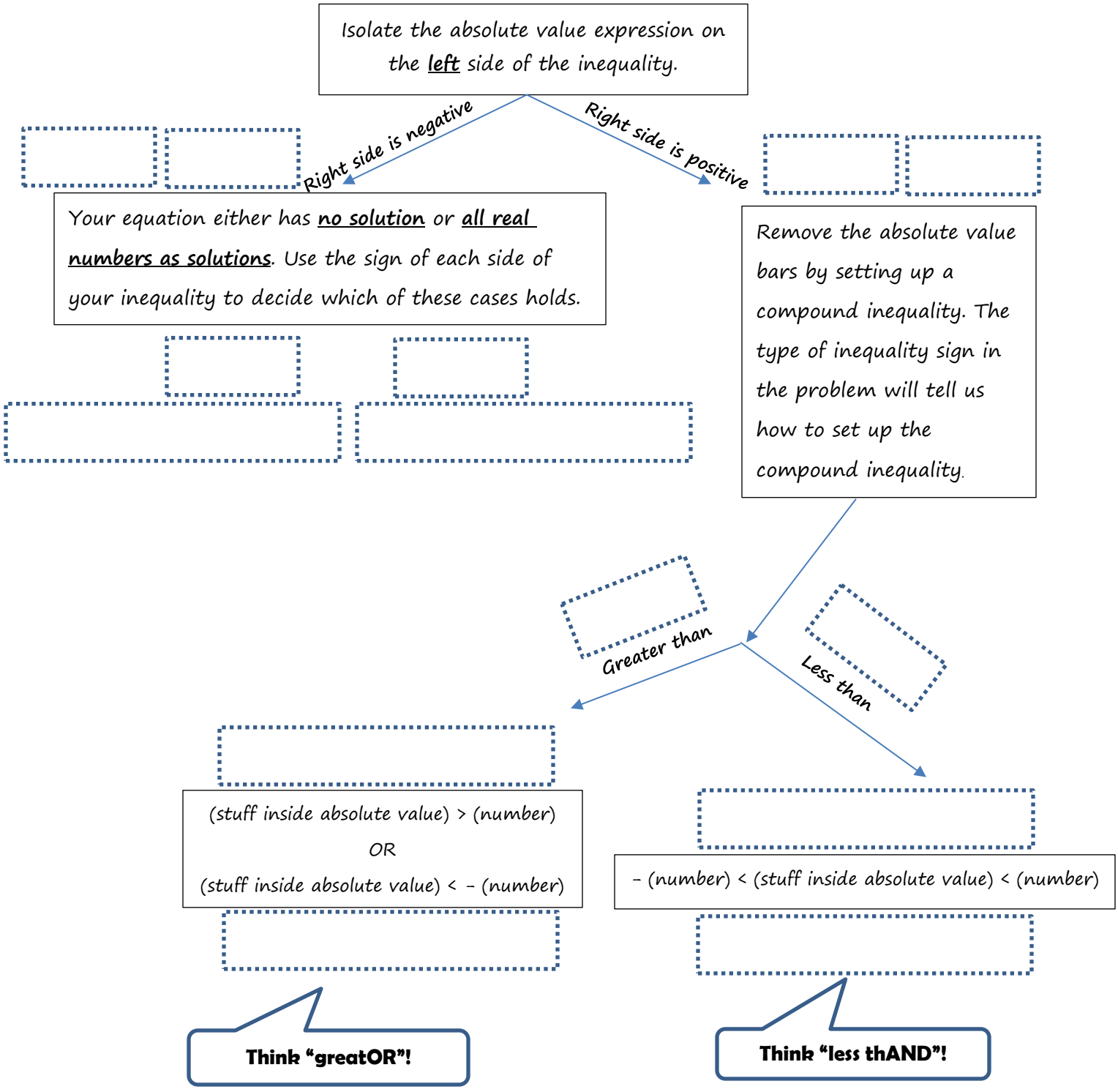
$\lvert x\rvert < 2$

$\lvert x\rvert < 2$

$\lvert x\rvert < -2$

$\lvert x\rvert < -2$

Tanja says that there is a better way to get the solutions, rather than just guessing. Below are her steps.



5. A. Follow Tanja's steps for the absolute value inequality $|x+2| \leq 3$. Be sure to show each step.

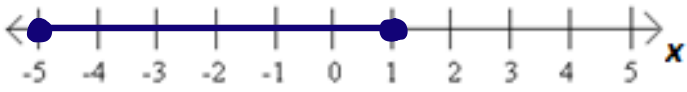
$|x+2| \leq 3$

$x+2 \leq 3$ $\quad$ $x+2 \geq -3$

$-2 \quad -2$ $\quad$ $-2 \quad -2$

$x \leq 1$ $\quad$ $x \geq -5$

B. Graph the solution set on the number line.



C. Does Tanya's steps work for $\leq$ or $\geq$ absolute value problems? Explain.

6. A. Follow Tanja's steps for the absolute value inequality $2|3x-1| < 20$. Be sure to show each step.

$|3x-1| < 10$

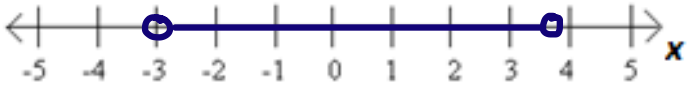
$3x-1 < 10$ $\quad$ $3x-1 > -10$

$+1 \quad +1$ $\quad$ $+1 \quad +1$

$3x < 11$ $\quad$ $3x > -9$

$x < \frac{11}{3}$ $\quad$ $x > -3$

B. Graph the solution set on the number line.

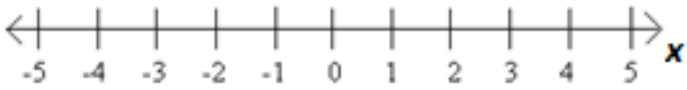


7. A. Follow Tanja's steps for the absolute value inequality $|2x+3|+6<5$. Be sure to show each step.

$|2x+3|<-1$

No Solution

B. Graph the solution set on the number line.

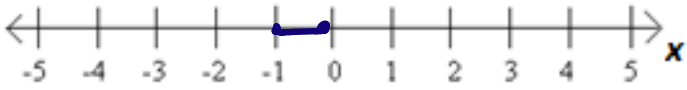


8. A. Follow Tanja's steps for the absolute value inequality $|3x+2|+6\geq 5$. Be sure to show each step.

$|3x+2|\geq -1$

All real numbers

B. Graph the solution set on the number line.



Lesson Summary

Solving Absolute Value Inequalities

$9|m-8|-10>26$ — Begin isolating the absolute value by adding 10 to both sides.

$9|m-8|>36$ — Isolate the absolute value by dividing by 9 on both sides.

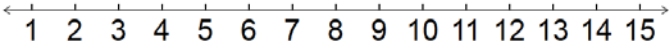
$|m-8|>4$ — Create a compound inequality.

$m-8>4$ or $m-8<-4$ — Solve the inequalities.

$m-8 > 4$ or $m-8 < -4$

$+8 \quad +8 \qquad +8 \quad +8$

$m > 12$ or $m < 4$



$9|m-8|-10<26$ — Begin isolating the absolute value by adding 10 to both sides.

$9|m-8|<36$ — Isolate the absolute value by dividing by 9 on both sides.

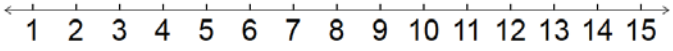
$|m-8|<4$ — Create a compound inequality.

$-4 < m-8 < 4$ — Solve the inequality.

$-4 < m-8 < 4$

$+8 \quad +8 \quad +8$

$4 < m < 12$



9. A. Draw a number line solution for each example in the Lesson Summary.

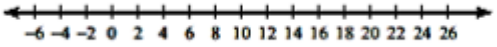
   B. Give the solutions in interval notation.

Homework Problem Set

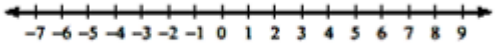
Solve and graph each absolute value inequality.

1. $|r| < 1$

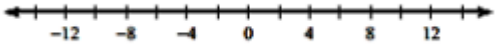
2. $|p - 10| < 15$



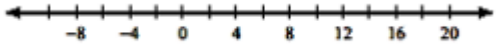
3. $|n| - 9 \geq -5$



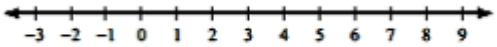
4. $-1 + \left|\frac{r}{10}\right| > 0$

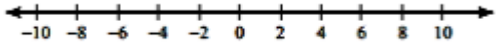
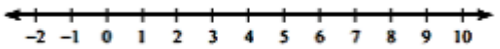


5. $|k - 5| > 13$



6. $\frac{|4 - 7n|}{3} > 2$



7. $-10\lvert p\rvert - 7 > -97$	8. $8\lvert -5 + 6m\rvert - 7 \le 49$
	

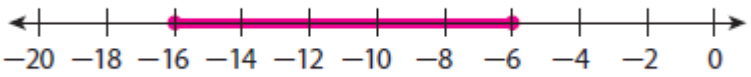
9. Lindsey is making some home-made toffee. The recipe says that she must bring the mixture to a boil at 285 degrees. If she is 7 degrees above or below, the toffee should turn out fine.

   Write, solve, and graph an absolute value inequality to model the range of temperatures that will make yummy toffee.

CHALLENGE PROBLEMS 10 – 12

10. Solve for x using the inequality $a\lvert x - b\rvert + c \le d$. Assume that $d - c > 0$.

11. Write a simple inequality with an absolute value symbol whose solution would be represented by the graph shown below.



12. A student made an error in the following problem. Determine where the error was made and then complete the problem correctly.

$$|x+3|+9<5$$

$$|x+3|<-4$$

$$4<x+3<-4$$

$$1< \quad x \quad <-7$$

Spiral Review

Combine Like Terms – [source: Open Middle]

13. **Open Ended** Using the whole numbers from 1 to 9 in the boxes below, create two expressions that are equivalent to one another. You can use each whole number at most once

$$\square x + \square x + \square + \square x = \square + \square x + \square$$

Solving Equations – [source: Open Middle]

14. **Open Ended** Use the whole numbers 1 through 9, at most one time each, to find the value of x closest to 0.

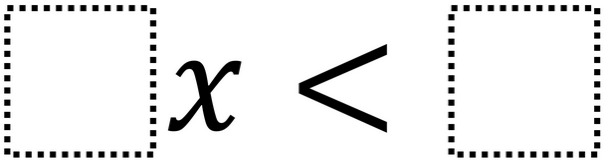
$$\frac{\square}{\square} x + \square = \square$$

15. **Open Ended** Use the digits 1 to 9, at most TWO times each, to fill in the boxes to make an equation with no solutions.

$$\square x + \square = \square x + \square$$

Inequalities – [source: Open Middle]

16. Using the integers – 4 to 4 at most one time each, create an inequality with solutions of $x > \frac{2}{3}$.



Spiral Review – Solving Equations

Solve each equation.

17. $10(-6 + n) = -150$

18. $51 = 5x - 9$

19. $6 = 3(8 + m)$

20. $-4 + 2(p - 8) = -36$

21. $2x + 4(5x - 3) = 5(4x + 4)$

22. $-3(4n + 1) = -6(2n - 6)$

Spiral Review – Solving Absolute Value Equations
[source: https://www.saylor.org/site/wp-content/uploads/2012/07/1.6-Absolute-Value-Practice.pdf]

23. $|x| = 8$

24. $|5 + 8a| = 53$

25. $|9n + 8| = 46$

26. $|3n - 2| = 7$

27. $|3 - x| = 6$

28. $-7|-3 - 3r| = -21$

29. $\frac{|-4 - 3n|}{4} = 2$

30. $8|x + 7| - 3 = 5$

31. $|x + 2| + 10 = 9$

32. $4|r + 7| + 3 = 59$