Lesson 10: Solving Inequalities

Opening Exercise

1. Find the solution set to each inequality. Express the solution graphically on the number line and give the solution in interval notation.

A. $x + 4 \leq 7$

$-4 \quad -4$

$x \leq 3$

B. $\frac{m}{3} + 8 > 9$

$-8 \quad -8$

$\frac{m}{3} > 1$

$m > 3$

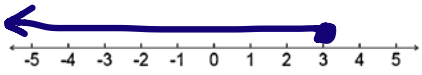
C. $8y + 4 < 7y - 2$

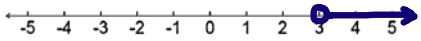
$-7y \quad -7y$

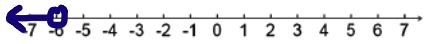
$y + 4 < -2$

$-4 \quad -4$

$y < -6$







D. $6(x - 5) \geq 30$

$6x - 30 \geq 30$

$+30 \quad +30$

$\frac{6x}{6} \geq \frac{60}{6}$

$x \geq 10$

E. $4(x - 3) > 2(x - 2)$

$4x - 12 > 2x - 4$

$-4x \quad -4x$

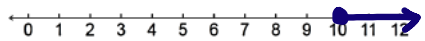
$-12 > -2x - 4$

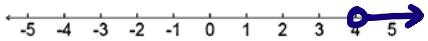
$+4 \quad +4$

$\frac{-8}{-2} > \frac{-2x}{-2}$

$4 < x$

$x > 4$





2. Stephanie says, "So far we have the following rules for inequalities:

If $A > B$, then $A + c > B + c$ for any real number c.

If $A > B$, then $kA > kB$ for any positive real number k."

Explain to your partner what Stephanie means by these statements. Be prepared to share out with the class.

Stephanie is quite clear in her rules that you cannot multiply by a negative number. Let's see what happens if we do multiply by a -1 with the inequality $5 - C > 2$.

$5 - C > 2$ → multiplied by -1 gives us the inequality → $-5 + C > -2$

3. Find one number that works for the first one. Does it work for the second one?

$5 - C > 2$

$5-2>2$

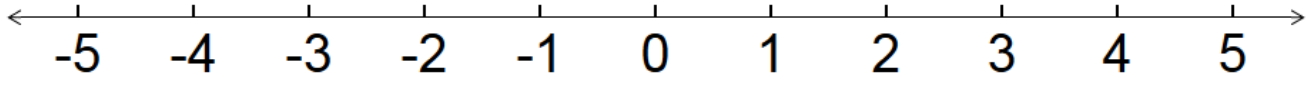
$3>2$

$-5 + C > -2$

$-5+2>-2$

-3 ~~>-2~~

4. A. Let's look at why this happens. If we choose two numbers on the number line, let's say 2 and 4 and mark them. We can see that $2 < 4$.



B. Now multiply our two numbers by -1 and mark these new numbers on the number line. You should now see that $-4 < -2$. What does multiplying by -1 do to our inequality?

The Properties of Inequalities

- Addition property of inequality: If $A > B$, then $A + c > B + c$ for any real number c.
- Multiplication property of inequality: If $A > B$, then $kA > kB$ for any positive real number k.

5. Use the properties of inequality to show that each of the following is true for any real numbers p and q.

A. If $p \geq q$, then $-p \leq -q$.

B. If $p < q$, then $-5p > -5q$.

6. Based on the results from Exercises 4 and 5, how might we expand the multiplication property of inequality?

- Multiplication property of inequality: If $A > B$, then $kA > kB$ for any positive real number k.
- Multiplication property of inequality: If $A > B$, then _____ for any negative real number k.

7. Find the solution set to each inequality. Express the solution graphically on a number line.

A. $-2f < -16$

$\frac{-2f}{-2} < \frac{-16}{-2}$

$f > 8$

B. $-\frac{x}{12} \leq \frac{1}{4}$

$x \geq \frac{1}{4}(-12)$

$x \geq -3$

C. $6 - a \geq 15$

$-6 \quad -6$

$\frac{-a}{-1} \geq \frac{9}{-1}$

$a \leq -9$

D. $-3(2x + 4) > 0$

$-6x - 12 > 0$

$+12 \quad +12$

$\frac{-6x}{-6} > \frac{12}{-6}$

$x < -2$

Lesson Summary

The Properties of Inequalities

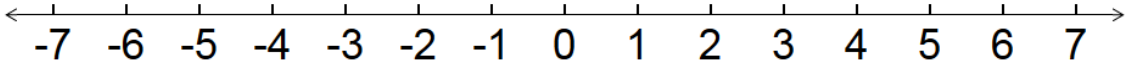
- Addition property of inequality: If $A > B$, then $A + c > B + c$ for any real number c.
- Multiplication property of inequality: If $A > B$, then $kA > kB$ for any positive real number k.

  If $A > B$, then $kA < kB$ for any negative real number k.

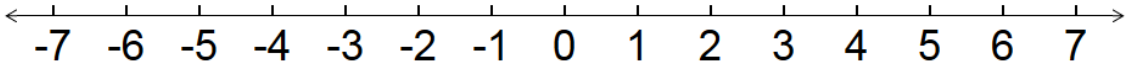
Homework Problem Set

Find the solution set to each inequality. Express the solution graphically on the number line and give the solution in interval notation.

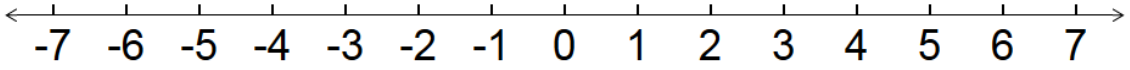
1. $2x < 10$



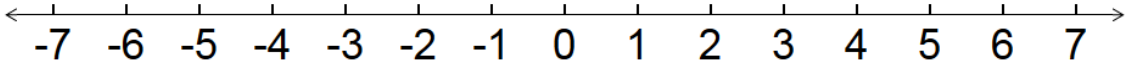
2. $-15x \geq -45$



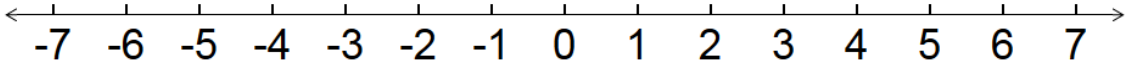
3. $\frac{2}{3}x \neq \frac{1}{2} + 2$



4. $-5(x - 1) \geq 10$



5. $13x < 9(1 - x)$



6. Solve $-\frac{x}{16}+1 \geq -\frac{5x}{2}$, for x without multiplying by a negative number. Then, solve by multiplying on both sides by -16.

7. Lisa brought half of her savings to the bakery and bought 12 croissants for $\$14.20$. The amount of money she brings home with her is more than $\$2.00$. Use an inequality to find how much money she had in her savings before going to the bakery. (Write the inequality that represents the situation, and solve it.)

8. Solve $-18 - 16t > -6 - 100t$, for t in two different ways: first without ever multiplying on both sides by a negative number and then by first multiplying on both sides by $-\frac{1}{2}$ or dividing by -2.