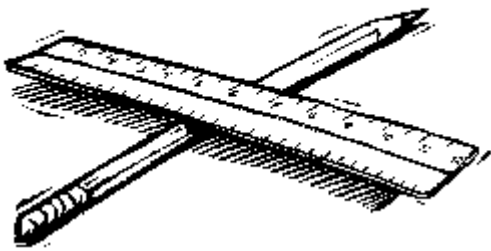


## Lesson 4: Describing the Center of a Distribution

In previous work with data distributions, you learned how to derive the mean and the median (the center) of a data distribution. In this lesson, we'll compare dot plots with their mean and median.

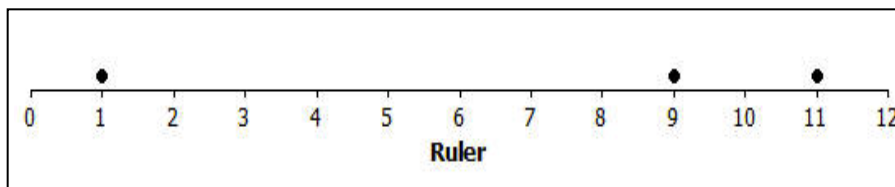
### Opening Exploration

In 6 – 8 grades you estimated and calculated a balance point of a data distribution, called the mean. Let's review what we learned about the balance point of a distribution.



The broad side of a pencil is placed underneath a 12-inch ruler to determine an approximate balance point of the ruler. Several quarters are taped to positions along the ruler.

Consider the following example of quarters taped to a lightweight ruler.



- Sam taped 3 quarters to his ruler. The quarters were taped to the positions 1 inch, 9 inches, and 11 inches. If the pencil was placed under the position 5 inches, do you think the ruler would balance? Why or why not?

No

- If the ruler did not balance, would you move the pencil to the left or to the right of 5 inches to balance the ruler? Explain your answer.

Move to the right

3. How do you think the quarter located at the 1-inch position affects the balance point? If that quarter were moved to the position of 3 inches, what would happen to the balance point?

At 3 in. it moves right

4. Is there any arrangement of the three quarters that you could make in which the balance point would be located at the position of 6 inches? Explain your arrangement and why you think it might work.

Yes, answers vary

5. How are the mean and the balance point related?

6. Estimate a balance point for the ruler. Complete the following based on the position you selected.

| Position of Quarter | Distance from Quarter to Your Estimate of the Balance Point |
|---------------------|---|
| 1                   | 6 5   |
| 9                   | 2 3   |
| 11                  | 4 5   |

balances at 7  
balances at 6

7. What is the sum of the distances to the right of your estimate of the balance point?

6 8

8. What is the sum of the distances to the left of your estimate of the balance point?

6 5

9. Do you need to adjust the position of your balance point? If yes, explain how.

10. Calculate the mean and the median of the position of the quarters. Does the mean or the median of the positions provide a better estimate of the balance point for the position of the 3 quarters taped to this ruler? Explain why you made this selection.

### Exploratory Challenge

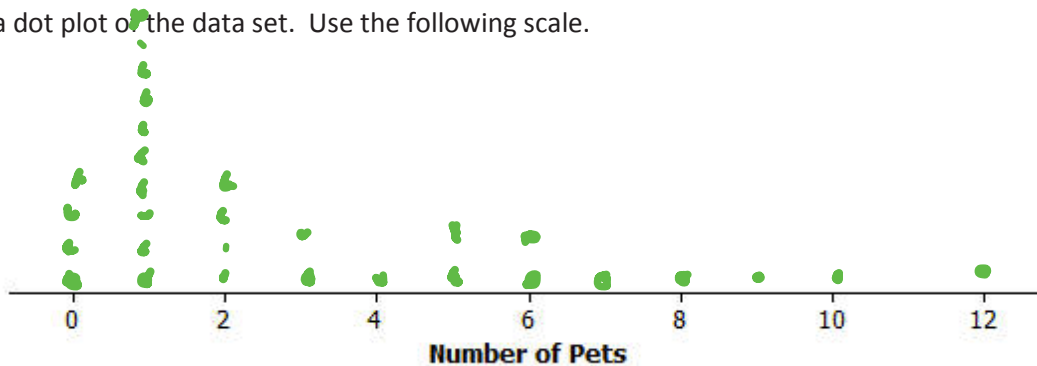
You will be considering three sets of data in this lesson and describing the center of distribution.

#### Data Set 1: Pet owners

Students from River City High School were randomly selected and asked, “How many pets do you currently own?” The results are recorded below.

|   |   |   |   |   |   |   |   |   |   |   |   |   |    |    |
|---|---|---|---|---|---|---|---|---|---|---|---|---|----|----|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1  | 2  |
| 2 | 2 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 6 | 7 | 8 | 9 | 10 | 12 |

11. Make a dot plot of the data set. Use the following scale.



12. Calculate the mean number of pets owned by the thirty students from River City High School. Calculate the median number of pets owned by the thirty students.

$$\frac{96}{30} = 3.2$$

$$\frac{2+2}{2} = \frac{4}{2} = 2$$

13. What do you think is a typical number of pets for students from River City High School? Explain how you made your estimate.

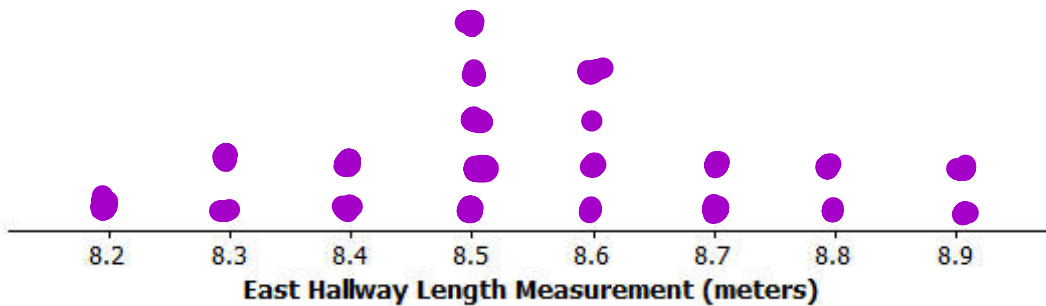
Median

**Data Set 2: Length of the east hallway at River City High School**

Twenty students were selected to measure the length of the east hallway. Two marks were made on the hallway's floor: one at the front of the hallway and one at the end of the hallway. Each student was given a meter stick and asked to use the meter stick to determine the length between the marks to the nearest tenth of a meter. The results are recorded below.

|     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 8.2 | 8.3 | 8.3 | 8.4 | 8.4 | 8.5 | 8.5 | 8.5 | 8.5 | 8.5 |
| 8.6 | 8.6 | 8.6 | 8.6 | 8.7 | 8.7 | 8.8 | 8.8 | 8.9 | 8.9 |

14. Make a dot plot of the data set. Use the following scale.



15. Why do you think that different students got different results when they measured the same distance of the east hallway?

The meter stick slipped. Didn't measure properly.

16. What is the mean length of the east hallway data set? What is the median length?

$$\frac{171.3}{20} = 8.57$$

$$\frac{8.5 + 8.6}{2} = \frac{17.1}{2} = 8.55$$

17. A construction company will be installing a handrail along a wall from the beginning point to the ending point of the east hallway. The company asks you how long the handrail should be. What would you tell the company? Explain your answer.

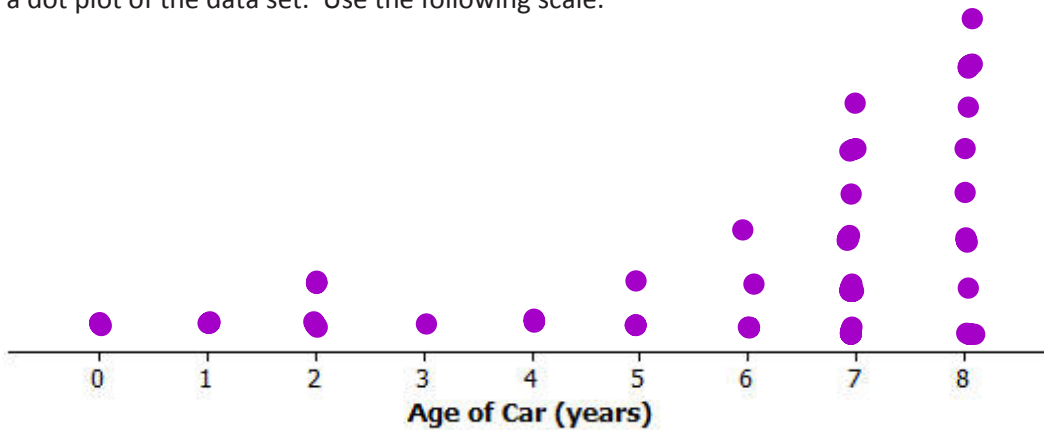
8.6

**Data Set 3: Age of cars**

Twenty-five car owners were asked the age of their cars in years. The results are recorded below.

|   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 6 | 6 | 7 | 7 |
| 7 | 7 | 7 | 7 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |   |

18. Make a dot plot of the data set. Use the following scale.



19. Describe the distribution of the age of cars.

20. What is the mean age of the twenty-five cars? What is the median age? Why are the mean and the median different?

$$\frac{146}{25} = 5.84$$

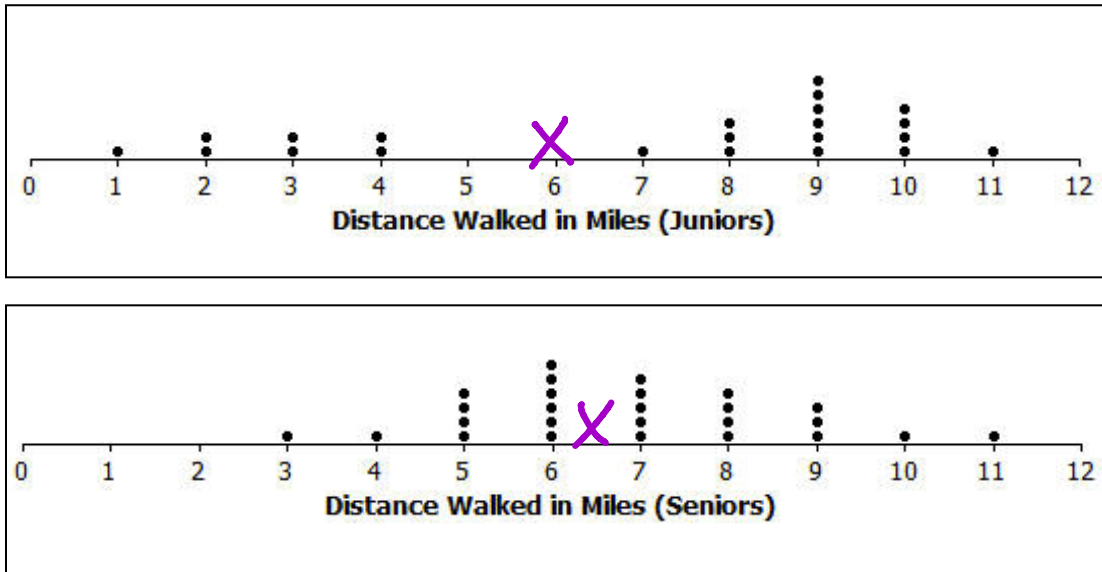
7 Skewed left

21. What number would you use as an estimate of the typical age of a car for the twenty-five car owners? Explain your answer.

The median because the data is skewed.

**Data Set 4: Distance Walked**

Twenty-two students from the junior class and twenty-six students from the senior class at River City High School participated in a walkathon to raise money for the school's band. Dot plots indicating the distances in miles students from each class walked are as follows.



22. Estimate the mean number of miles walked by a junior, and mark it with an X on the junior class dot plot. How did you estimate this position?

23. What is the median of the junior data distribution?

$$\frac{8+9}{2} = \frac{17}{2} = 8.5$$

24. Is the mean number of miles walked by a junior less than, approximately equal to, or greater than the median number of miles? If they are different, explain why. If they are approximately the same, explain why.

Less than because the data is skewed left.

25. How would you describe the typical number of miles walked by a junior in this walkathon?

8.5 (the median)

26. Estimate the mean number of miles walked by a senior, and mark it with an X on the senior class dot plot. How did you estimate this position?

27. What is the median of the senior data distribution?

$$\frac{7+7}{2} = \frac{14}{2} = \boxed{7}$$

28. Estimate the mean and the median of the miles walked by the seniors. Is your estimate of the mean number of miles less than, approximately equal to, or greater than the median number of miles walked by a senior? If they are different, explain why. If they are approximately the same, explain why.

Approx. equal

29. How would you describe the typical number of miles walked by a senior in this walkathon?

7 (both are the same)

30. A junior from River City High School indicated that the number of miles walked by a typical junior was better than the number of miles walked by a typical senior. Do you agree? Explain your answer.

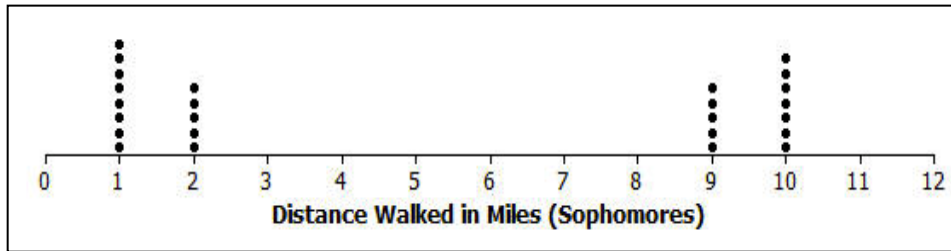
## Lesson Summary

- A dot plot provides a graphical representation of a data distribution, helping us to visualize the distribution.
- The mean and the median of the distribution are numerical summaries of the center of a data distribution.
- The mean of a data distribution represents a balance point for the distribution. The sum of the distances to the right of the mean is equal to the sum of the distances to the left of the mean.
- The mean of a set of data is determined by adding all of the data points and dividing by the number of data points.
- The median of a set of data is the middle number when the data points are placed in ascending or descending order. If two points are in the middle then the average of these two numbers is used as the median.
- When the distribution is nearly symmetrical, the mean and the median of the distribution are approximately equal. When the distribution is not symmetrical (often described as skewed), the mean and the median are not the same.



**Homework Problem Set**

The twenty-five sophomores who participated in the walkathon reported their results. A dot plot is shown below.



1. What is different about the sophomore data distribution compared to the data distributions for juniors and seniors?

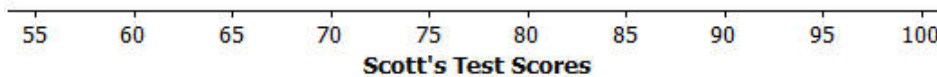
2. Estimate the balance point of the sophomore data distribution.

3. What is the median number of miles walked by a sophomore?

4. How would you describe the sophomore data distribution?

Consider another example of balance. Mr. Jackson is a mathematics teacher at Waldo High School. Students in his class are frequently given quizzes or exams. He indicated to his students that an exam is worth 4 quizzes when calculating an overall weighted average to determine their final grade. During one grading period, Scott got an 80% on one exam, a 90% on a second exam, a 60% on one quiz, and a 70% on another quiz.

How could we represent Scott's test scores? Consider the following number line.



5. What values are represented by the number line?
6. If one “•” symbol is used to represent a quiz score, how might you represent an exam score?
7. Represent Scott's exams and quizzes on this number line using “•” symbols.
8. Mr. Jackson indicated that students should set an 85% overall weighted average as a goal. Do you think Scott met that goal? Explain your answer.
9. Place an X on the number line at a position that you think locates the balance point of all of the “•” symbols. Determine the sum of the distances from the X to each “•” on the right side of the X.
10. Determine the sum of the distances from the X to each “•” on the left side of the X.

11. Do the total distances to the right of the X equal the total distances to the left of the X?

12. Based on your answer to Problem 11, would you change your estimate of the balance point? If yes, where would you place your adjusted balance point? How does using this adjusted estimate change the total distances to the right of your estimate and the total distances to the left?

13. Scott's weighted average is 81. Recall that each exam score is equal to 4 times a quiz score. Show the calculations that lead to this weighted average.

14. How does the calculated mean score compare with your estimated balance point?

15. Compute the total distances to the right of the mean and the total distances to the left of the mean. What do you observe?

16. Did Scott achieve the goal set by Mr. Jackson of an 85% average? Explain your answer.

Consider the following scenario.



The company that created a popular video game “Leaders” plans to release a significant upgrade of the game. Users earn or lose points for making decisions as the leader of an imaginary country. In most cases, repeated playing of the game improves a user’s ability to make decisions. The company will launch an online advertising campaign, but at the moment, they are not sure how to focus the advertising. Your goal is to help the company decide how the advertising campaign should be focused. Five videos have been proposed for the following target audiences:

Video 1: Target females with beginning level scores

Video 2: Target males with advanced level scores

Video 3: Target all users with middle range level scores

Video 4: Target males with beginning level scores

Video 5: Target females with advanced level scores

17. Why might the company be interested in developing different videos based on user scores?

18. Thirty female users and twenty-five male users were selected at random from a database of people who play the game regularly. Each of them agreed to be part of a research study and report their scores. A leadership score is based on a player's answers to leadership questions. A score of 1 to 40 is considered a beginning level leadership score, a score of 41 to 60 is considered a middle level leadership score, and a score of greater than 60 is considered an advanced level leadership score.

Use the following data to make a dot plot of the female scores, a dot plot of the male scores, and a dot plot of the scores for the combined group of males and females.

**Female scores:**

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 10 | 20 | 20 | 20 | 30 | 30 | 30 | 40 | 40 | 40 |
| 50 | 50 | 55 | 65 | 65 | 65 | 65 | 65 | 70 | 70 |
| 70 | 70 | 76 | 76 | 76 | 76 | 76 | 76 | 76 | 76 |

**Male scores:**

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 15 | 20 | 20 | 25 | 25 | 25 | 25 | 30 | 30 | 30 |
| 30 | 30 | 30 | 35 | 35 | 35 | 35 | 35 | 40 | 40 |
| 40 | 45 | 45 | 45 | 50 |    |    |    |    |    |



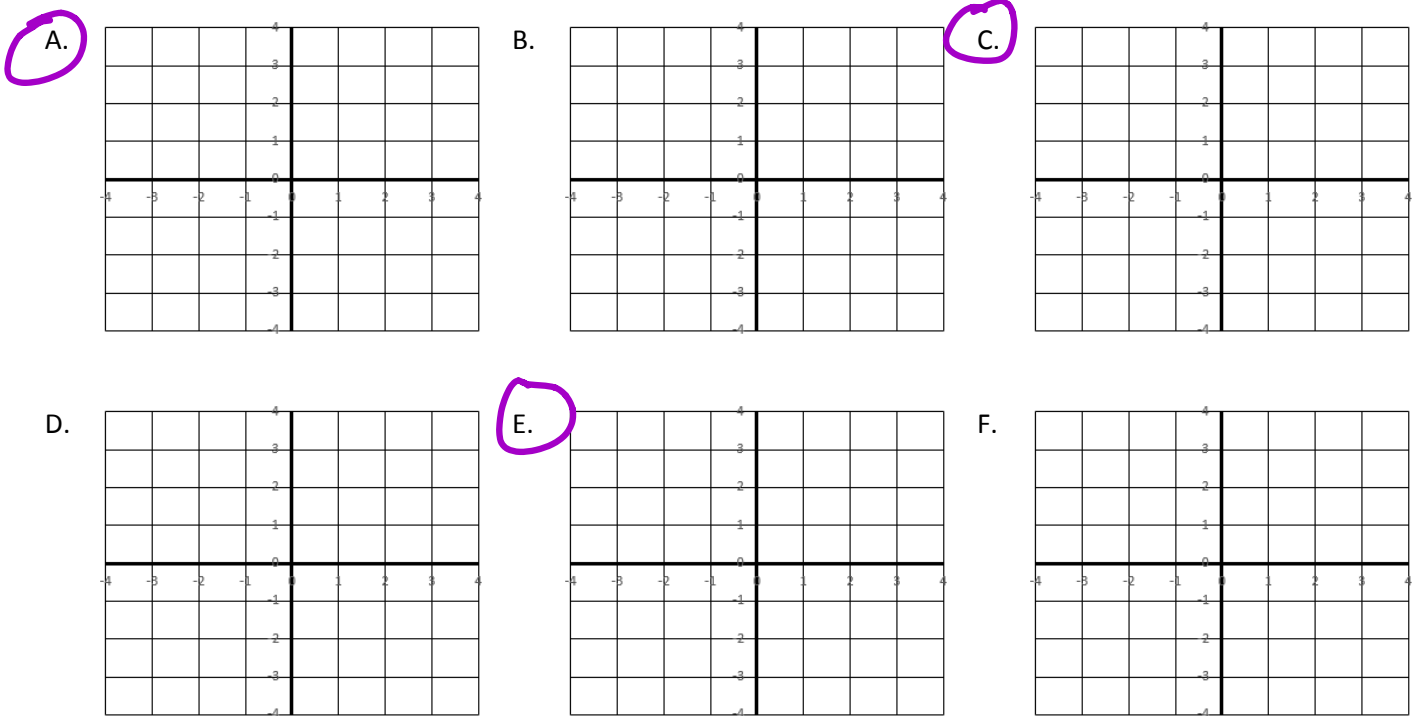
19. What do you think is a typical score for a female user? What do you think is a typical score for a male user? Explain how you determined these typical scores.
20. Why is it more difficult to report a typical score for the overall group that includes both the males and females?
21. Production costs will only allow for two video advertisements to be developed. Which two videos would you recommend for development? Explain your recommendations.

**Spiral Review – Slope and Writing Equations of Lines**

22. Determine the slope between each pair of coordinates.

- A. (0, 0) and (2, 3)
- B. (0, 0) and (-2, 3)
- C. (0, 0) and (2, -3)
- D. (0, 0) and (-2, -3)
- E. (0, 0) and (0, 3)
- F. (0, 0) and (2, 0)

23. Graph each set of coordinates from Exercise 6 and check if the slopes you found were correct.



24. Write the equation of each line for the graphs in Exercise 7. Use the formula  $y = mx + b$ , where  $m$  = slope and  $b$  =  $y$ -intercept.

- A.  $y =$  \_\_\_\_\_
- B.  $y =$  \_\_\_\_\_
- C.  $y =$  \_\_\_\_\_
- D.  $y =$  \_\_\_\_\_
- E.  $y =$  \_\_\_\_\_
- F.  $y =$  \_\_\_\_\_

