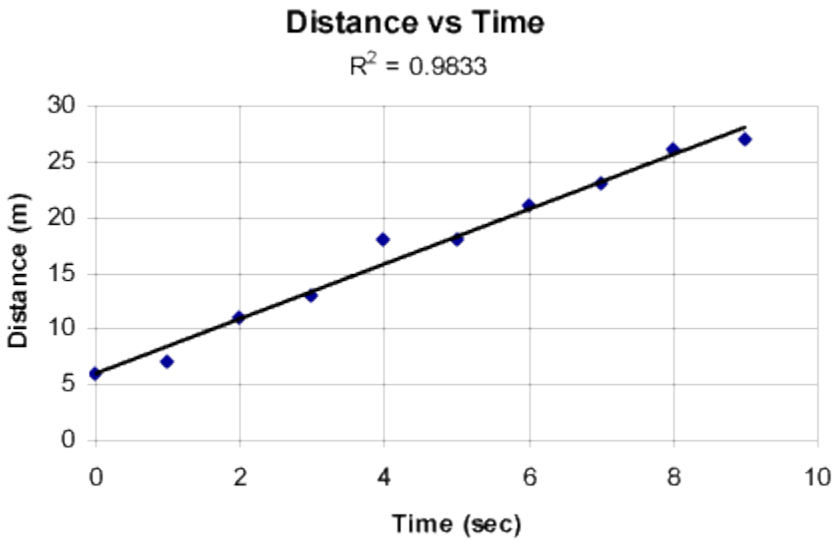
Lesson 12: Lines of Best Fit

In Lesson 11, you graphed and analyzed a variety of scatterplots. In this lesson, you'll how to write the equation of a line to describe the scatterplot. This equation will allow you to make predictions about the data.



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Opening Exercise - Writing an Equation of a Line

The equation of a line can be in the form $y = mx + b$, where m represents the slope of the line and b represents the y-intercept. We'll start with a strong positive linear scatterplot. The line of best fit has been drawn in for you.

1. Choose two points on the line of best fit and then determine the slope between these two lines.

   A. My Points: (0 6) and (2 11)

   B. Slope using my two points: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-6}{2-0} = \frac{5}{2} = 2.5$ m/sec

   C. What does this slope mean in terms of the data?

   Move 2.5m every 1 sec.

2. What is the estimated y-intercept for the line?

   6

3. Write the equation of the best fit line using $y = mx + b$, where m = slope and b = y-intercept.

   $y = 2.5x + 6$

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4. The slope found using the actual data points and a calculator or computer program was 2.5. How much did your slope differ from this value? Why do you think there might be a difference?

It was the same.

5. The *y*-intercept generated using the actual data points and a calculator or computer program was 6. How much did your *y*-intercept differ from this value? Why do you think there might be a difference?

It was the same.

6. You can use your equation to find other points on the line.

   A. Use the graph to estimate the distance when time is 7.5 seconds.

$y = 2.5(7.5) + 6$ $y = 2.5x + 6$

$y = 18.75 + 6 = \boxed{24.75}$

   B. Use your equation from Exercise 3 to estimate the distance when time is 7.5 seconds.

   C. Which model (graph or equation) would you use to estimate the distance at 10 seconds? What is that value?

   D. How much time would you estimate would go by for 15 meters? Which model did you use? Why?

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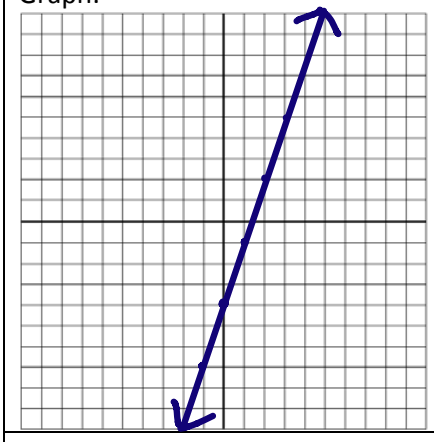
Practice – Writing & Graphing Equations of Lines

7. Slope = 3; *y*-intercept = -4

Equation: $y = 3x - 4$

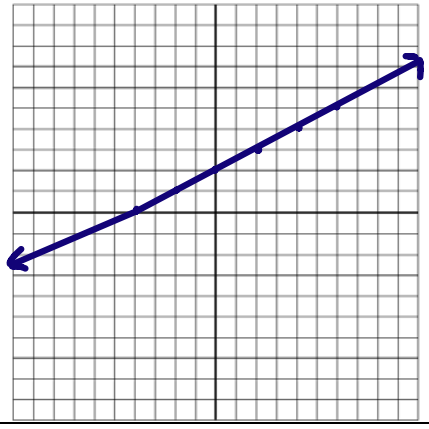
Graph:



8. Slope = ½; *y*-intercept = 2

Equation: $y = \frac{1}{2}x + 2$

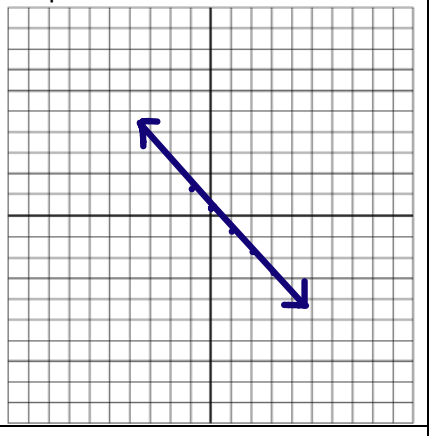
Graph:



9. Slope = -1; *y*-intercept = ¼

Equation: $y = -x + \frac{1}{4}$

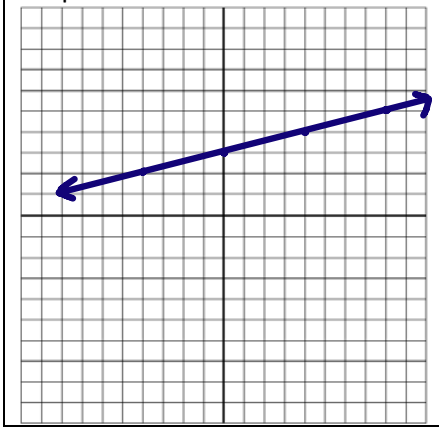
Graph:



10. Two points on the line (4, 4) and (8, 5) with a *y*-intercept of 3. $\frac{5-4}{8-4} = \frac{1}{4}$

Equation: $y = \frac{1}{4}x + 3$

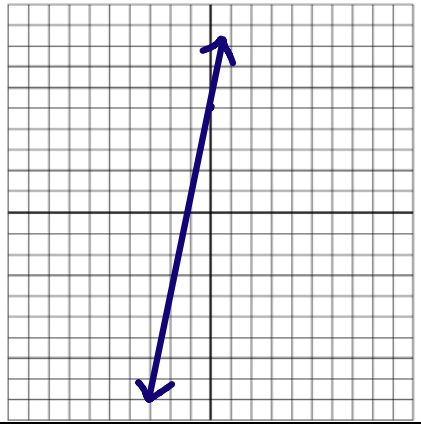
Graph:



11. Two points on the line (-3, -9) and (0, 5) with a *y*-intercept of 5. $\frac{5-(-9)}{0-(-3)} = \frac{5+9}{3} = \frac{14}{3}$

Equation: $y = \frac{14}{3}x + 5$

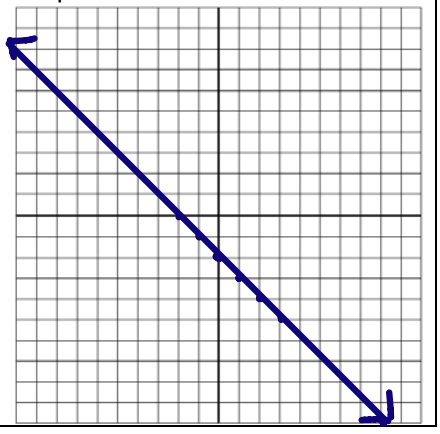
Graph:



12. Two points on the line (-2, 0) and (4, -6) with a *y*-intercept of -2. $\frac{-6-0}{4-(-2)} = \frac{-6}{6} = -1$

Equation: $y = -x - 2$

Graph:



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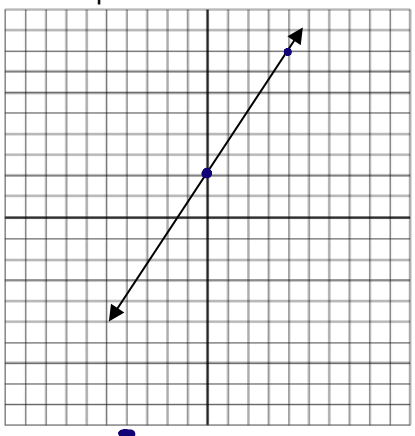
Practice – Writing Equations of Lines from Graphs

13. Graph:

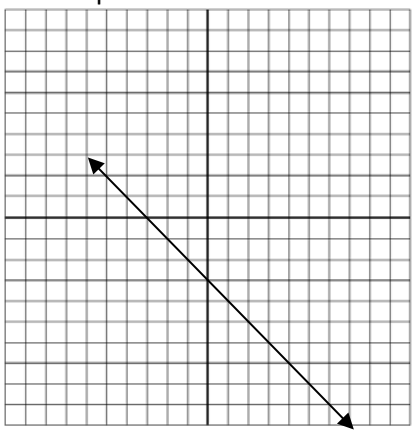


Slope = $\frac{3}{2}$

y-intercept = 2

Equation: $y=\frac{3}{2}x+2$

14. Graph:

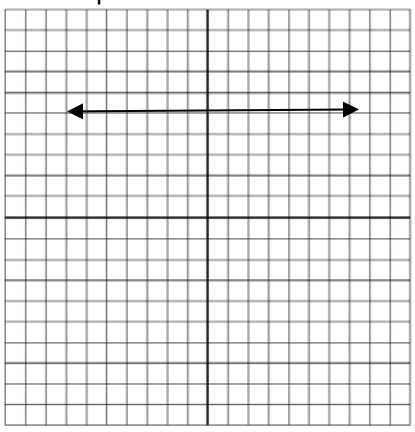


Slope = -1

y-intercept = -3

Equation: $y=-x-3$

15. Graph:

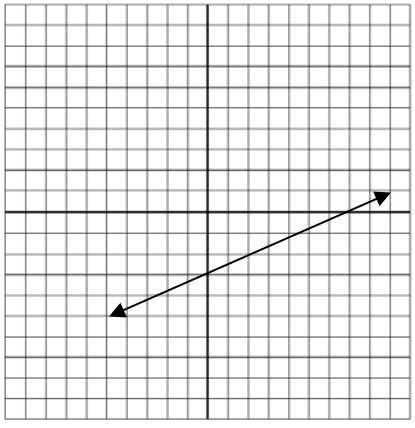


Slope = 0

y-intercept = 5

Equation: $y=5$

17. Graph:

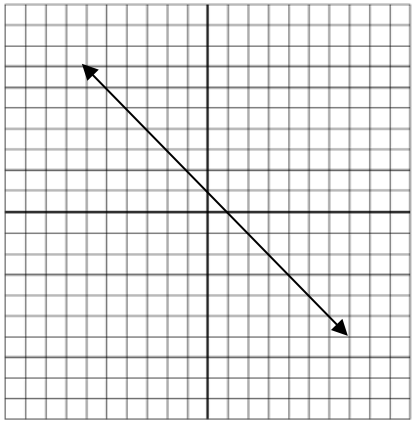


Slope = $\frac{1}{2}$

y-intercept = -3

Equation: $y=\frac{1}{2}x-3$

18. Graph:

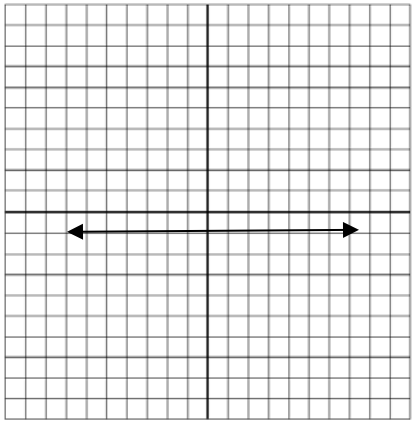


Slope = -1

y-intercept = 1

Equation: $y=-x+1$

19. Graph:



Slope = 0

y-intercept = -1

Equation: $y=-1$

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Point-Slope Form of a Line

The equation you wrote in Exercise 3 is in slope-intercept form or $y = mx + b$. Point-slope is another very useful form of a linear equation. For point-slope we need any point on the line and the slope, not just the y-intercept and the slope of the line. The general form looks like $y - y_1 = m(x - x_1)$ where (x_1, y_1) is any point on the line and m is the slope of the line.

20. Suppose two points on the line in the Opening Exercise are (1, 8.5) and (7, 23.5).

A. Use the point-slope equation to write the equation of the line. $\frac{23.5-8.5}{7-1} = \frac{15}{6} = \frac{5}{2} = 2.5$

Find the slope: 2.5

Use the formula. You may use either point for the equation: $y - 8.5 = 2.5(x - 1)$

B. Discuss with your partner how you could rearrange this equation to isolate the y and get it in slope-intercept form.

C. Rewrite your equation in Part A in slope-intercept form.

$y - 8.5 = 2.5x - 2.5$

$+8.5 \qquad +8.5$

$y = 2.5x + 6$

D. How does this equation compare to the one you wrote in Exercise 3?

When no y-intercept is available, we can use a different form of the equation of a line. This new form comes from the slope formula.

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} \quad \textbf{or} \quad m = \frac{y_2 - y_1}{x_2 - x_1} \ (x_2 - x_1)$$

Point-Slope Equation of a Line: $y_2 - y_1 = m(x_2 - x_1)$ **or** $y - y_1 = m(x - x_1)$

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21. A. How did they get from the slope equation to the point-slope equation?

B. What information do you need to find the equation of a line? List all possibilities.

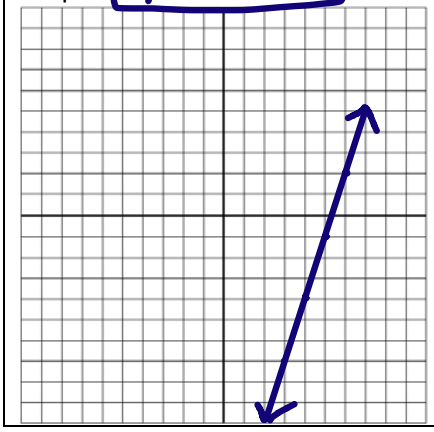
If I have . . .	Then I can use . . .
slope and y-int.	$y=mx+b$
slope and a point or 2 points	$y-y_1=m(x-x_1)$

Practice Writing and Graphing Linear Equations in Point-Slope Form Given the Slope and one Point

22. Slope = 3; Point = (3, -7)

Equation: $y+7=3(x-3)$ $y+7=3x-9$ $y=3x-16$

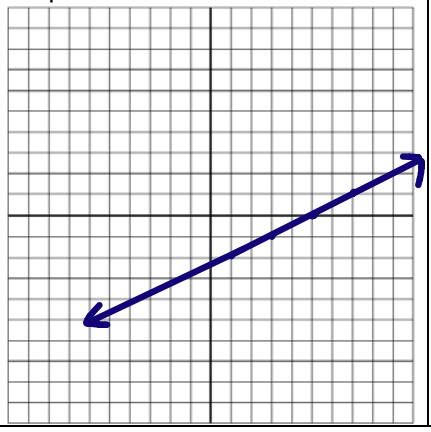
Graph:



23. Slope = ½; Point = (5,0)

Equation: $y-0=\frac{1}{2}(x-5)$ $y=\frac{1}{2}x-2.5$

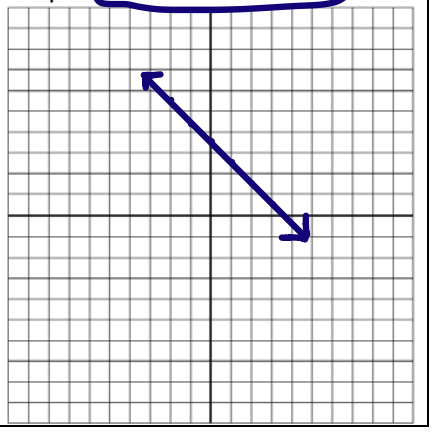
Graph:



24. Slope = -1; Point = (- ½ , 4)

Equation: $y-4=-1(x+\frac{1}{2})$ $y-4=-x-\frac{1}{2}$ $y=-x+3.5$

Graph:



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Practice Writing and Graphing Linear Equations in Point-Slope Form Given two Points

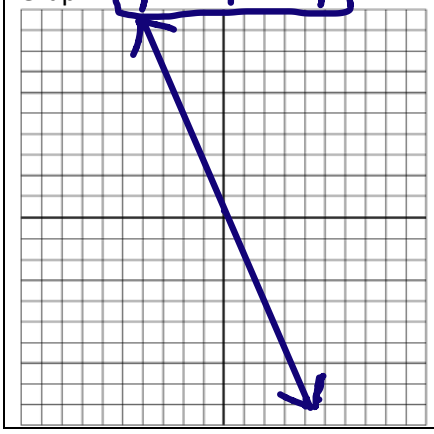
25. Point = (-1, 2); Point = (3, -7)

Slope: $\frac{-7-2}{3+1} = \frac{-9}{4}$

Equation: $y-2=-\frac{9}{4}(x+1)$

$y-2=-\frac{9}{4}x-\frac{9}{4}$

Graph: $y=-\frac{9}{4}x+\frac{1}{4}$

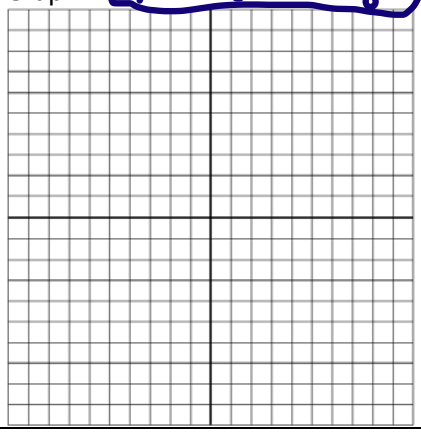


26. Point = (-3, 5); Point = (5,0)

Slope: $\frac{0-5}{5+3} = \frac{-5}{8}$

Equation: $y-0=-\frac{5}{8}(x-5)$

Graph: $y=-\frac{5}{8}x+\frac{25}{8}$

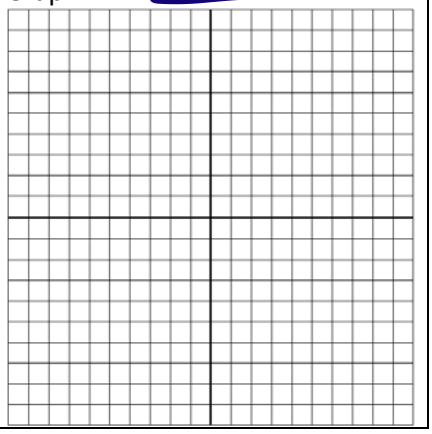


27. Point = (8, 0); Point = (4 , 4)

Slope: $\frac{4-0}{4-8} = \frac{4}{-4} = -1$

Equation: $y-0=-1(x-8)$

Graph: $y=-x+8$



Lesson Summary

Writing Equations of Lines

- If you have two points on your line you can use the **Point-Slope Equation** given by $y-y_1=m(x-x_1)$, where m = slope and (x_1, y_1) is a point on the line.
- If you have one point and the slope of your line you can use the **Slope-Intercept Equation** given by $y = mx + b$, where m = slope and b = y-intercept.

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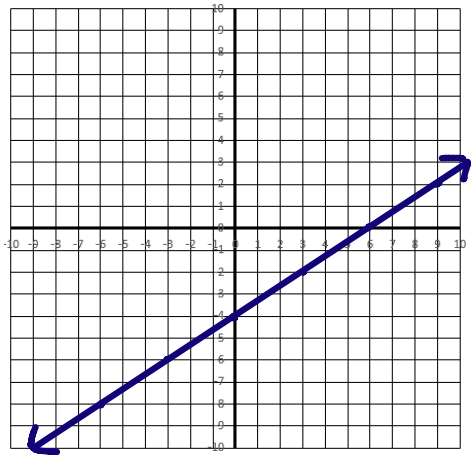
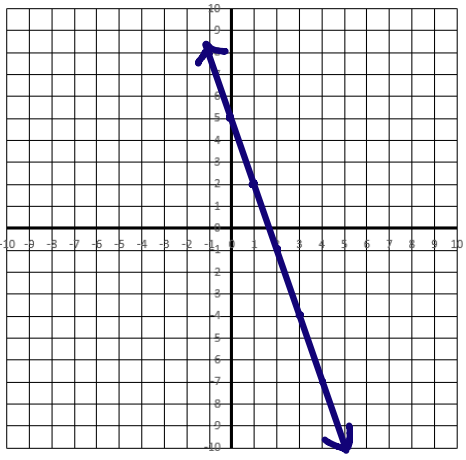
Homework Problem Set

1. Graph the following lines.

A. $y = \frac{2}{3}x - 4$

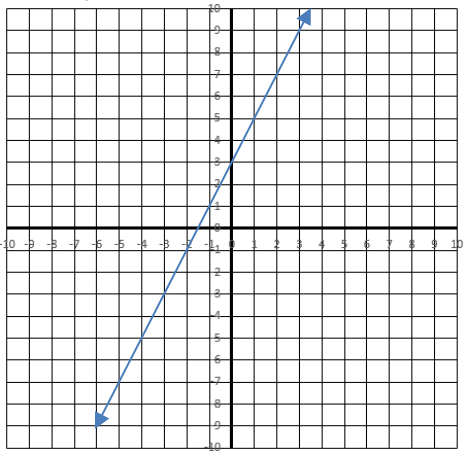


B. $y = -3x + 5$

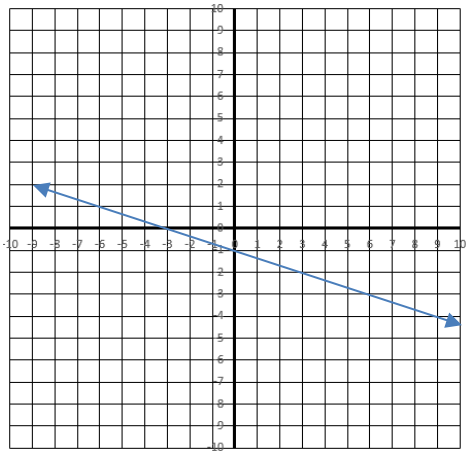


2. Write the equation of the line in slope-intercept form.

A. $y = 2x + 3$



B. $y = -\frac{1}{3}x - 1$



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3. A. Why is $y = mx + b$ called "slope-intercept" form and $y - y_1 = m(x - x_1)$ is called "point-slope" form?

you are given the slope and y-int.

you are given the slope and a point

B. If you wanted to change an equation from point-slope to slope-intercept form, what would you do?

4. Change $y - 3 = -2(x + 1)$ to slope-intercept form.

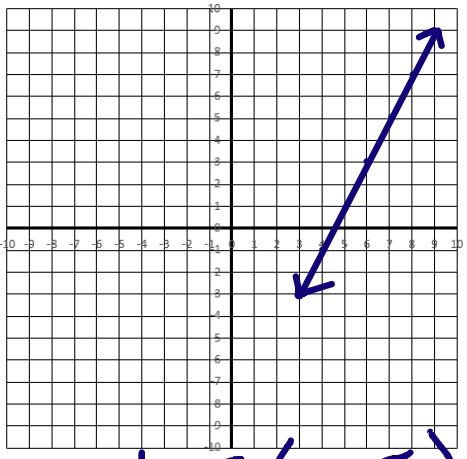
$y - 3 = -2x - 2$

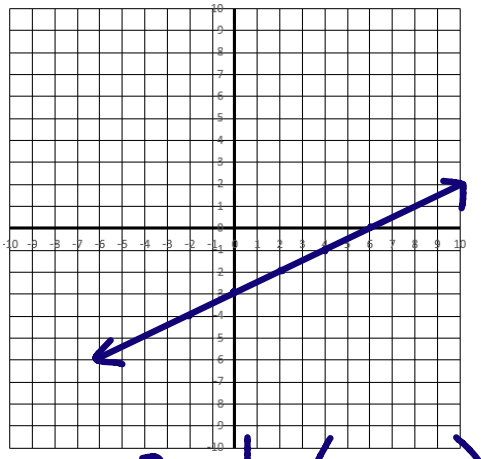
$y = -2x + 1$

5. Write the equation of the line in point-slope form. Then change the equation to slope-intercept form and graph the equation to see if your calculations were correct.

A. (5, 1) and $m = 2$ $y - 1 = 2(x - 5)$ $y - 1 = 2x - 10$ $y = 2x - 9$

B. (2, -2) and (10, 2) $\frac{2+2}{10-2} = \frac{4}{8} = \frac{1}{2}$ $y + 2 = \frac{1}{2}(x - 2)$ $y + 2 = \frac{1}{2}x - 1$ $y = \frac{1}{2}x - 3$





Point-Slope: $y - 1 = 2(x - 5)$ $y + 2 = \frac{1}{2}(x - 2)$

Slope-Intercept: $y = 2x - 9$ $y = \frac{1}{2}x - 3$

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6. Write the equation in both point-slope and slope-intercept forms.

A. (2, 5) and (5, -1)

$\frac{-1-5}{5-2} = \frac{-6}{3} = -2$

$y-5=-2(x-2)$

Point-slope form: $y-5=-2(x-2)$

Slope-intercept form: $y=-2x+9$

B. (0, 5) and $m=\frac{1}{3}$

Point-slope form: $y-5=\frac{1}{3}(x-0)$

Slope-intercept form: $y=\frac{1}{3}x+5$

7. Write the equation of the line represented by the data in the table. Write your answer in slope-intercept and point-slope forms.

x	-2	-1	0	1	2	3
y	9	7	5	3	1	-1

Point-slope form: _____

Slope-intercept form: _____

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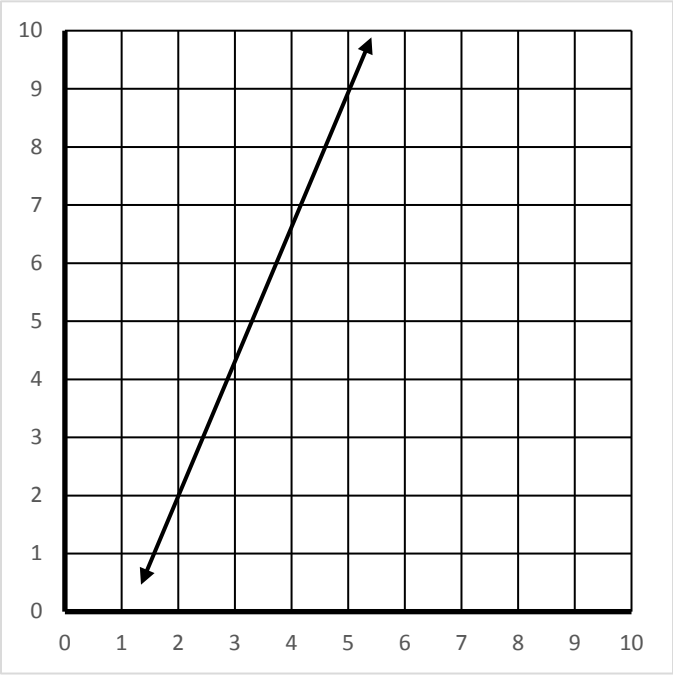
8. Write the equation of the line in point-slope and slope-intercept forms.



Point-slope form: _____

Slope-intercept form: _____

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