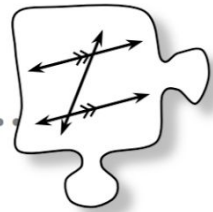


# 9.1.3 What if the angle is on the outside?

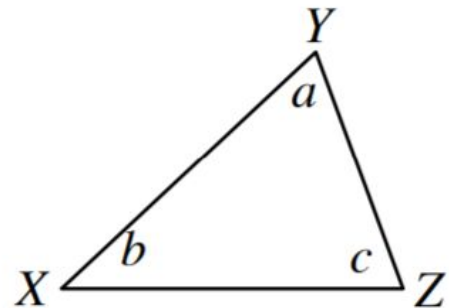
## Exterior Angles in Triangles



So far in this section, you have investigated angle relationships in situations with parallel lines and within triangles. In this lesson, you will continue to look at angle relationships. This time, you will investigate the angle relationships on the inside and outside of a triangle.

**9-27.** Use the diagram at right to name each of the indicated angles below using three letters. Reread the Math Notes box in Lesson 9.1.1 if you need to remember how to do this.

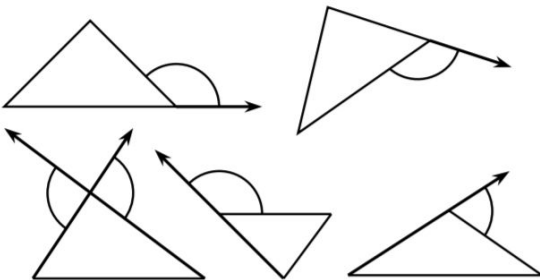
- a.  $\angle a$   $\angle ZYX$        $\angle XYZ$
- b.  $\angle b$   $\angle ZXY$        $\angle YXZ$
- c.  $\angle c$   $\angle XZY$        $\angle YZX$



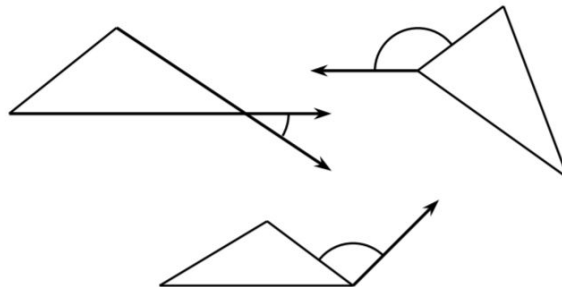
**9-28.** Read the information below. Then follow the directions that follow

**Exterior angles** are formed by extending a side of the triangle. The two angles across the triangle from the exterior angle are called **remote interior angles**. In each figure located below part (a),  $\angle A$  and  $\angle B$  are remote interior angles with respect to exterior angle  $\angle BCD$ .

These are exterior angles:



These are NOT exterior angles:



9-28 a. In each figure located below part (a),  $\angle A$  and  $\angle B$  are remote interior angles with respect to exterior angle  $\angle BCD$ .

- Calculate the missing angle measures in each figure and record them in the table.
- Look for a pattern in the relationship between the measure of  $\angle BCD$  (the exterior angle) and the sum of the measures of  $\angle A$  and  $\angle B$  (the remote interior angles).

(Note that  $m\angle BCD$  means the measure of  $\angle BCD$ . Similarly,  $m\angle A$  means the measure of  $\angle A$ .)

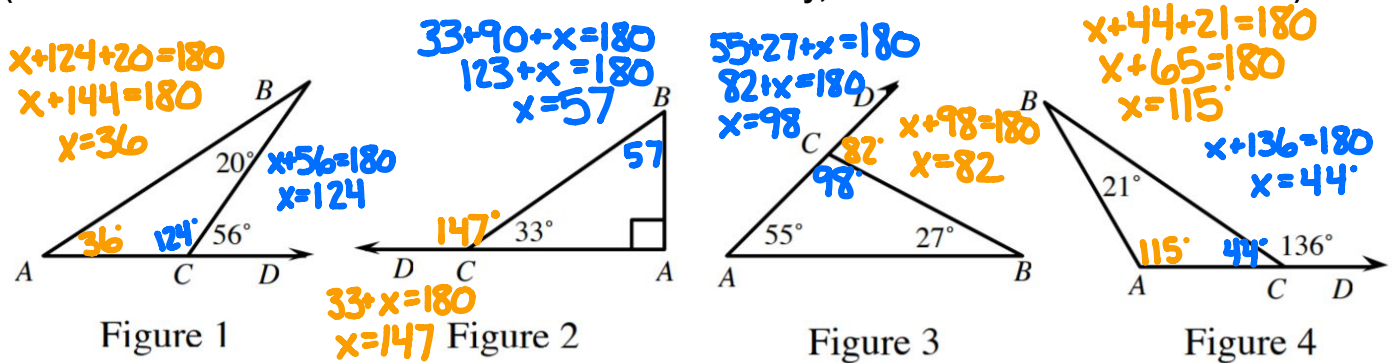


Figure Number	$m\angle A$	$m\angle B$	$m\angle ACB$	$m\angle BCD$	$m\angle A + m\angle B$
1	$36^\circ$	$20^\circ$	$124^\circ$	$56^\circ$	$56^\circ$
2	$90^\circ$	$57^\circ$	$33^\circ$	$147^\circ$	$147^\circ$
3	$55^\circ$	$27^\circ$	$98^\circ$	$82^\circ$	$82^\circ$
4	$115^\circ$	$21^\circ$	$44^\circ$	$136^\circ$	$136^\circ$

9-28 b. Compare your results for  $m\angle BCD$  (the exterior angle) and the sum of  $m\angle A$  and  $m\angle B$  (the remote interior angles) for each figure. Discuss your observations with your team.

$$m\angle A + m\angle B = m\angle BCD$$

9-28 c. Write a conjecture about the relationship of an exterior angle to the two remote interior angles.

The sum of the remote interior angles = the exterior angle.

9-29. Calculate the measures of the angles requested. Each part is a separate problem.

a. If  $m\angle 1 = 53^\circ$  and  $m\angle 2 = 71^\circ$ , find  $m\angle 4$ .

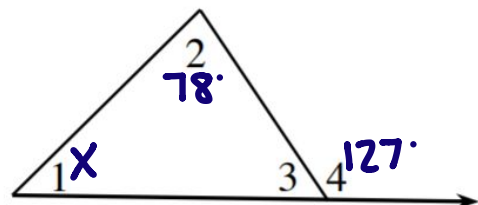
$$53 + 71 = x$$

$$x = 124^\circ$$

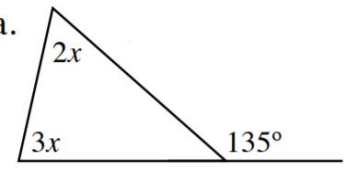
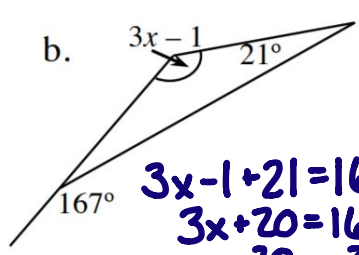
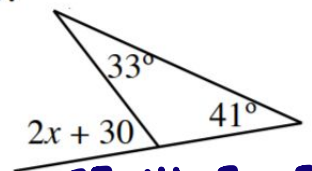
b. If  $m\angle 2 = 78^\circ$  and  $m\angle 4 = 127^\circ$ , find  $m\angle 1$ .

$$x + 78 = 127$$

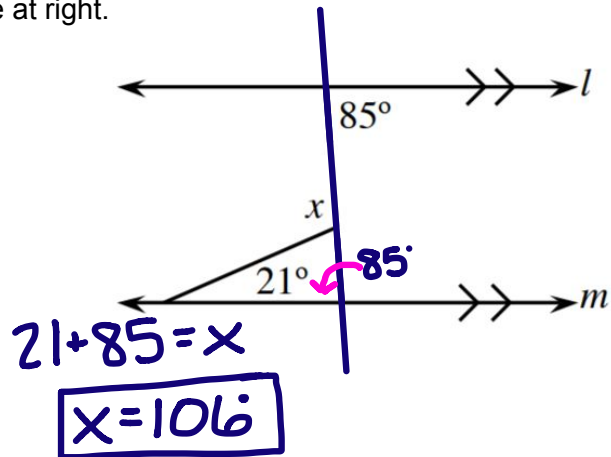
$$\begin{array}{r} x + 78 = 127 \\ -78 \quad -78 \\ \hline x = 49 \end{array}$$



9-30. Use your conjecture from part (c) of problem 9-28 to solve for x in each figure below.

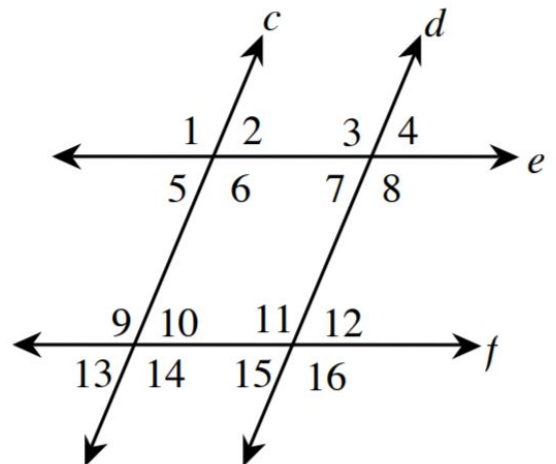
<p>a.</p>  $2x + 3x = 135$ $5x = 135$ $\frac{5x}{5} = \frac{135}{5}$ $x = 27$ <p>x = <u>27</u></p>	<p>b.</p>  $3x - 1 + 21 = 167$ $3x + 20 = 167$ $\begin{array}{r} 3x + 20 = 167 \\ -20 \quad -20 \\ \hline 3x = 147 \\ \frac{3x}{3} = \frac{147}{3} \\ x = 49 \end{array}$ <p>x = <u>49</u></p>	<p>c.</p>  $33 + 41 = 2x + 30$ $74 = 2x + 30$ $\begin{array}{r} 74 = 2x + 30 \\ -30 \quad -30 \\ \hline 44 = 2x \\ \frac{44}{2} = \frac{2x}{2} \\ x = 22 \end{array}$ <p>x = <u>22</u></p>
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9-31. Additional Challenge: Solve for x in the figure at right.



9-33. Based on the given information, determine which pairs of lines, if any, are parallel. If none are necessarily parallel, write "none."

- a.  $m\angle 2 = m\angle 7$  **c and d**
- b.  $m\angle 3 = m\angle 11$  **e and f**
- c.  $m\angle 1 = m\angle 12$  **none**
- d.  $m\angle 13 = m\angle 12$  **c and d**
- e.  $\angle 6$  and  $\angle 7$  are supplementary. **c and d**



9-34. In each angle problem below, solve for the variable(s). Write the names of the definition(s) and relationship(s) that justify the steps in your solution.

a.

$$\begin{aligned}
 2x + 3 + 132 &= 180 \\
 2x + 135 &= 180 \\
 -135 \quad -135 & \\
 \hline
 2x &= 45 \\
 \frac{2x}{2} \quad \frac{45}{2} & \\
 \hline
 x &= 22.5
 \end{aligned}$$

b.

$$\begin{aligned}
 49 + 90 + e &= 180 \\
 139 + e &= 180 \\
 -139 \quad -139 & \\
 \hline
 e &= 41
 \end{aligned}$$

$$\begin{aligned}
 131 + d &= 180 \\
 -131 \quad -131 & \\
 \hline
 d &= 49
 \end{aligned}$$

c.

$$\begin{aligned}
 19 + 24 &= f \\
 \hline
 f &= 43
 \end{aligned}$$

## LESSON SUMMARY



MATH NOTES

## METHODS AND MEANINGS

### Exterior Angle Theorem for Triangles

An **exterior angle** of a triangle is an angle outside of the triangle created by extending one of the sides of the triangle. In the diagram at right,  $\angle 4$  is an exterior angle. The **Exterior Angle Theorem for Triangles** states that the measure of the exterior angle of a triangle is equal to the sum of the **remote interior angles**. In the diagram,  $\angle 1$  and  $\angle 2$  are the remote interior angles to  $\angle 4$ . Note that some texts call these angles “opposite interior angles.” In symbols:

$$m\angle 4 = m\angle 1 + m\angle 2$$

